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of
The Institute of Radio Engineers

EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

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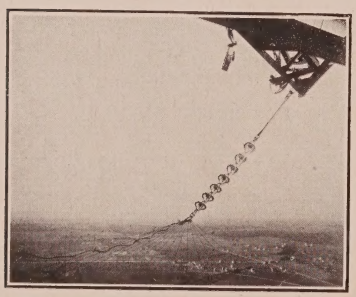

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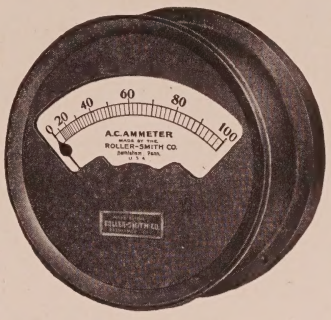

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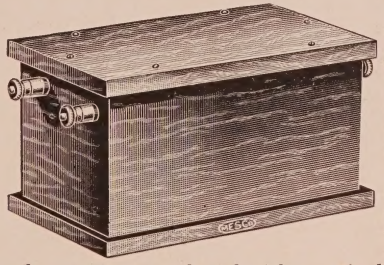

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The Institute of Radio Engineers
(INCORPORATED)

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The Membership of the Standing Committees of The Institute for 1917 will be announced in an early issue of The Proceedings of The Institute of Radio Engineers.

PROCEEDINGS OF THE SECTIONS

WASHINGTON SECTION

A meeting of the Washington Section of The Institute of Radio Engineers was held at the University Club, Washington, on the evening of Monday, November 27, 1916. Prior to the meeting, a dinner was given in honor of the retiring Chairman of the Section, Colonel Samuel Reber, U. S. A. There were thirty-two members present. In the course of the meeting, Lieutenant-Colonel George O. Squier, U. S. A., was elected Chairman for 1917, and the remaining members of the Executive Committee of the Washington Section were re-elected.

BOSTON SECTION

On the evening of Thursday, November 2, 1916, a meeting of the Boston Section was held in the Cruft High Tension Laboratory, Harvard University. Professor A. E. Kennelly, President of the Institute, presented a paper on "Telephone Receivers" illustrated by lantern slides.

On the evening of Thursday, December 14, 1916, a meeting of the Boston Section was held at the Cruft High Tension Laboratory. Mr. H. J. W. Fay presented an illustrated paper on "Submarine Signaling."

SEATTLE SECTION

On the evening of September 9, 1916, a meeting of the Seattle Section of the Institute was held in Seattle. A paper by Mr. Robert H. Marriott, Past-President of the Institute, on "Radio Shadows" was presented. The attendance was ten. Certain financial arrangements were carried out on the same occasion.

SAN FRANCISCO SECTION

A meeting of the San Francisco Section of the Institute was held at the Engineers' Club, Mechanics Institute Building, San Francisco, on the evening of November 21, 1916; Mr. W. W. Hanscom presiding. A paper on "The Proposed Navy Bill and Government Ownership of Radio Stations" was presented by Mr. George S. de Sousa. There were twenty-nine members present. Following the paper, Messrs. Hanscom and Greaves gave discussions thereon. Matters dealing with the further organization of the San Francisco Section were then considered by the meeting.

A dinner and meeting of the San Francisco Section were held on the evening of December 19, 1916, at the Engineers' Club,

San Francisco, Mr. W. W. Hanscom presiding. At the dinner the attendance was twenty, and at the meeting following the attendance was forty-nine. Two papers were presented at the meeting. The first of these, by Mr. Oscar C. Roos, was on "Radio Conditions in the Philippine Islands." The second, by Mr. Ellery W. Stone, dealt with "Additional Experiments with Impulse Excitation." The first paper was discussed by Messrs. Hanscom and Greaves, and the second by Messrs. Pratt, F. C. Ryan, and the Author. Local organization and financial matters were further considered at the meeting.

ENGINEERING PRECAUTIONS IN RADIO INSTALLATIONS*

By

Robert H. Marriott

(Expert Radio Aid, Navy Yard, Bremerton, Washington)

Probably all devices used to produce some desirable result may, under certain conditions, produce or contribute to the production of undesirable results, or damage. The probability of damage from radio apparatus compares favorably with that from other useful devices, and is apparently decreasing. However, radio apparatus may produce damage, and a discussion of the matter may result in future prevention of damage.

In this paper the subject will be considered under four general headings.

1. Wherein dangerous shocks may be received from radio apparatus.
2. Wherein radio apparatus provides a path for currents other than radio currents.
   (a) Lightning.
   (b) Antennas coming into contact with lighting or power lines.
3. Wherein radio apparatus provides the current or potential by direct discharge.
4. Wherein radio apparatus provides the current or potential by induction.

1. Injurious shocks may be received from the transmitter circuits used in very high power stations or in lower power stations should the operator come in contact with the power transformer secondary when the transformer is disconnected from the radio circuits. Radio frequency currents are usually, at worst, only disagreeable.

There are, or were, a few cases of dangerous practice along these lines. One was to shunt the operating key, so that the transformer secondary was at a fairly high potential when the key was open. Another dangerous method and probably by

*Delivered before The Institute of Radio Engineers, New York, June 2, 1915.

far the most dangerous to the life of the operator, was to use alternating current primary generators which gave an open circuit voltage as high as 500 or 600 volts and connecting that high voltage circuit thru the operating key.

Possibly it is reasonably safe to use a generator open circuit voltage as high as 250 but, all things considered, it may be best to bring this voltage down nearer 110, even if efficiency of transformation has to be sacrificed slightly.

2 (a). The danger of fire being produced by lightning striking the antenna is apparently less than the danger in ways mentioned under headings 2 (b), 3, and 4. Personally, I have never seen lightning strike an antenna, nor have I seen evidence that lightning has struck an antenna. However, I have frequently seen antennas discharge to ground when lightning apparently struck at some distant point. For example, in one case, using an antenna 200 feet (60 meters) high, the discharge jumped a gap of 3.5 inches (8.7 cm.) to the ground. On several occasions, in mountainous districts, I have seen lightning striking apparently on all sides of a radio station. On one such occasion, lightning struck a one-story house about six blocks from an antenna 200 feet (60 meters) high. On another such occasion, lightning apparently struck a high tension line near the radio station, judging from the crash which was apparently coincident with the flash and from the fact that the high tension transformer in the sub-station within a couple of hundred feet was burned out.

2 (b). At one time a report was brought in that lightning had struck a radio station burning up the receiving apparatus. On investigation it was found that someone had changed the antenna wires from their former position and had placed them across and above a 1,200 volt line. When the ropes supporting the antenna stretched, the antenna dropped down on the 1,200 volt line and grounded this line thru the receiving apparatus, burning up the receiving apparatus. On other occasions, antenna wires have dropped across telephone lines, and lighting and power circuits. In the case of the telephone lines, the radio transmitters discharged to the telephone line, usually short-circuiting the telephone lightning arresters; while in the case of the lighting and power circuits, the power circuits usually were short circuited, burning out fuses. However, in those cases, had the receiving instrument been connected, it is possible that the power circuits might have discharged thru the receiving instruments and burned them.

In cities, where lighting, power and telephone circuits are exposed, trouble may arise from antennas dropping across such wires, and in the larger cities where fire alarm circuits and telephone circuits frequently run across the roofs of houses, these circuits may be frequently damaged by antenna wires dropping across them and by their receiving direct discharges from the transmitters.

The greatest number of fires I have noticed starting from direct discharge of transmitters have been where roof insulators or deck insulators leaked current to the roof or deck, and where the roof or deck was of some combustible material. However, none of these fires have resulted in serious conflagrations, probably because they almost invariably occurred during rain or very damp weather, the dampness or rain serving both to short-circuit the insulator and put out the fire.

Portions of transmitters, such as condenser supports, transformer supports, etc., have frequently been charred to some extent. There is less danger of fire being caused by the apparatus which is mainly in use now because, with the exception of auxiliary apparatus as used by one company but now being discontinued, the plain antenna method of connection of the transmitter has been discontinued. This plain antenna connection brings the full spark gap potential to the roof or deck insulator, thereby causing it to break down. A majority of the cases observed where the roof or deck was set on fire were brought about by this type of apparatus.

For the benefit of persons who have not given thought to the subject of insulating radio transmitters, a few points concerning insulation may be proper. These points refer mainly to the transmitter and include the antenna.

A. Air is a good insulator. Its insulating qualities are least liable to be affected by dust, moisture, or age; also, it is cheap. That is, it is desirable to use plenty of air space, when practicable, between points where a discharge might take place.

B. Long and narrow surface insulation is desirable, much on the same principle that a long, narrow conductor has a higher resistance than a short, thick one.

C. Insulators having corrugated surfaces, or surfaces which furnish tortuous paths, are desirable, as such insulators require radio frequency currents to travel over long paths. For the same reasons, such insulators are desirable for direct current and audio frequency potentials.

D. Non-combustible, non-absorbent materials (for example,

porcelain) are preferable for insulators where it is possible to use them.

E. Insulator surfaces should be kept clean and dry.

In the earlier days of radio work, a common method of bringing the antenna thru the wall of the house was to bring this connection thru the middle of a large window pane. This practice was usually fairly satisfactory and not very expensive.

For inside work, the writer adopted a general rule of providing surface insulation equal to eight times the sparking distance thru air. For example, if the wire used in the circuit would spark to objects at a distance of one inch (2.5 cm.) thru air, this wire was held away by a porcelain rod one inch (2.5 cm.) in diameter and eight inches (20. cm.) long.

Porcelain cleats in series are probably as inexpensive an insulator as may be used for guying small antennas, considering their insulating qualities.

4. For the purpose of this paper, the currents which are set up in conductors not connected to antenna, but due to the radio frequency currents in that antenna, will be referred to as "induced radio currents", and the transference of energy from the antenna to other unconnected circuits will be referred to as "by radio induction".

The greatest damages from fire which is known to me have occurred where the transmitters were not connected with the point which took fire. In these cases the transmitter caused high potentials in conductors which were more or less distant from the transmitter; that is, these conductors acted somewhat as receiving antennas, and were close enough to rise to a high potential. Where these conductors consisted of telephone circuits the lightning arresters provided on the telephone circuits usually short circuited to ground by the fusing of the metal in the arrester.

This grounding of the telephone circuits usually rendered the telephone circuit inoperative. In the cases of lighting and power circuits carrying direct current or alternating current, such as 60 cycle alternating current, the high potential radio frequency alternating current induced on these lines was apparently superimposed on the direct current or audio frequency alternating current. The radio frequency current produced on these lines was frequently of very high voltage comparatively while the other current (direct, or audio frequency) on the lines was of comparatively high amperage. When the radio potential occurred at a point within striking distance of an object at op-

posite potential, it apparently discharged and carried the direct current or audio frequency current over after it. In many cases the arcs so formed held until the terminals of the arc or part of the circuit burned away. Power transformers, lighting transformers, motors, generators, relays, magnetos, watt meters, ammeters, volt meters, lamp sockets, rosettes, etc., burned out or were rendered inoperative apparently from this cause. On a number of occasions lamp cord carrying 110 volt direct current, or 60 cycle alternating current, has been short circuited, and on one occasion an 8 foot (2.4 meters) drop cord disappeared in flame and a nearby motor was short circuited. On other occasions, lamp cord lying against wooden moulding short circuited and burned, setting fire to the wooden moulding. On these occasions, people were nearby and put the fire out before it reached any material magnitude.

On one occasion receiving and transmitting apparatus were located very near to the transmitter. The result was that motor and generator windings, relay windings, reactance coil windings, etc., were repeatedly short circuited. This was stopped by providing radio frequency paths thru condensers across points which developed high radio frequency potential; also, by placing the wiring in grounded iron conduit, and the short sections of wiring of the switchboard in grounded lead covered wires; and finally, by placing a grounded wire netting screen between the transmitter and the apparatus. All of these expedients were put into effect before noticeable potentials were avoided.

Radio frequency currents possibly in some cases have been superimposed on high tension circuits of the transformers, at least across portions of the secondary of such transformers. It is not quite so easy to conceive how this radio frequency potential may occur in the secondary where so many turns of fine wire are used.* However, when transformers were placed in certain relation and near radio frequency circuits they broke down sometimes between sections and sometimes from secondary to primary, and similar transformers when substituted and moved further away or turned at an angle did not break down.

In the United States in 1901, in order to prevent induction in mast guys, these guys were made of rope. In 1902, owing to the stretching and contraction of the rope in dry and wet weather, the writer substituted steel guys with rope blocks and falls at

*A probable explanation is the distributed capacity of the secondary windings and consequent internal resonance effects with breakdown.

the bottom of the guys to serve both as insulators and as means for adjusting the guys. About this time, or before, others used wooden strain insulators in the guys. On some occasions both the rope insulators and the wooden strain insulators were burned by current leakage between the guys and ground. Even on shipboard, attempts were made at times to insulate guys and stays between masts. However, owing to the difficulty of providing insulation which would not leak, the principle of thoroly grounding the stays and guys was adopted. Stays and guys and other metallic conductors, such as hand rails, occasionally discharged to passengers, causing considerable excitement and fear on the part of some steamship companies that passengers might be electrocuted. The remedy used for this was thoroly to ground stays and guys, etc. Even the metal whistle cords on vessels occasionally discharged to damp wood work, etc., and often a person who tried to manipulate the whistle received a shock. These were grounded by using flexible wire ground connection. Steel beams, steam pipes, long bolts, anchor chains, and other conducting materials, on vessels, have been known to spark to ground or to other conductors. Conduits containing electrical wiring have apparently discharged to the ends of wiring where the conduits were not grounded. Metal roofs in the vicinity of land stations, and metal roofs of wharves, have discharged to ground, causing charring of the wood to such an extent that fear of fire resulted.

On account of sparking on their vessels, one line had a tendency to accuse the radio apparatus of being responsible for nearly anything that went wrong with the electrical circuits on the vessel, even going so far as to say that the radio currents went down thru the vessel and into the water condenser of the engine and caused electrolysis to such an extent that the water condenser had to be replaced!

On a line where the vessels were almost entirely constructed of wood, sparking, charring, and injured apparatus resulted at a number of points. The mast stays were wrapped with houslin and passed thru thimbles connected to the hull of the vessel, thereby insulating the mast stay from the hull of the vessel by the houslin. This houslin was set on fire and burned away, due to the sparking between the mast stays and the hull of the vessel via the thimble. On these vessels the mast head lights, running lights, and port and starboard lights, were connected to the pilot house signal light switch board by means of rubber covered twin conductors without metal covering. All of these signal light

circuits were burned out from time to time due to sparking across the lines or between the lines and ground. Annunciator circuits and call bell circuits thruout the vessels discharged to metal portions of the ship, and in some cases caused slight charring of woodwork.

On one occasion a steamship company asked that their vessel be gone over with a view to preventing any possibility of igniting explosives which they expected to carry. In this case it was recommended that all metallic conductors in the hold and in the vicinity of the hold be thoroly grounded and electrically connected together, even the short metal ladders and supports which extended from one deck to another.

Three instances are recalled of wooden masts set on fire due to the discharging of guys to each other thru the woodwork of the mast. In two of the cases the masts were burned off several feet from the top. In these cases the guys were 50 feet (15 meters) or more from the antennas.

It has been found that radio currents were induced in the metallic paint on masts and on some occasions the metal paint was removed and a portion of the mast varnished. Some years ago it was the rule to make all radio masts of wood. Also wooden top masts have been required on shipboard because of the radio apparatus.

Regarding the ability of sparks to start fire, that obviously depends on the heat developed by the spark and the heat required by the combustible material. Very small sparks are almost universally used for igniting gas or gasolene vapor in gas engines, and it is quite possible that similar gas might be ignited by equally small or smaller sparks on shipboard or at other points near radio stations. Sparks developed by radio transmitters might be capable of igniting oils such as are found, for example, in the paint lockers on vessels. Theoretically, radio might cause distress conditions by setting the ship on fire and then relieve these conditions by bringing aid!

While the paper has been confined practically entirely to personal observations, the conclusion is not to be drawn that the damaging results always occur. The instances mentioned practically cover all the cases noted during a period of about 15 years use of radio frequency circuits, including radio frequency apparatus operated under a large variety of relations to adjacent conductors at stations on both coasts of the United States, at numerous points inland, and on the vessels of several nations.

Protection against radio frequency currents of dangerous potential being induced in low potential direct current of audio frequency circuits may be brought about to a considerable extent by taking advantage of the ways in which radio frequency currents differ from direct current and audio frequency currents.

Condensers of small capacity impede radio frequency currents very much less than audio frequency currents; (that is, radio frequency currents usually find an easy path thru small condensers, while practically no 60 cycle current or direct current will flow thru small condensers.) For practical purposes small condensers may be assumed to be good conductors for radio frequency currents and insulators for direct current and alternating current having frequencies in the neighborhood of 30 to 500 cycles.

Condensers have been installed in series with fuses to ground. This practice is objectionable because if the fuses burn out, the lines are left unprotected at a time when such protection is most likely to be needed, and unless the fuses are in some way arranged to notify some person, it is quite probable that they will not be known to have burned out until after damage occurs to the low tension circuits.

Mica condensers in which lead foil was used have been found to provide automatic self fusing devices without destroying the service ability of the condenser; for example, when a sheet of mica punctured making a small hole, the lead foil melted away from around the hole until the arc was extinguished and the condenser then operated as before.

Radio frequency currents do not penetrate very far into the conductor, or flow to any great extent in a conductor when that conductor is screened by a concentric conductor such that the radio frequency may flow in the concentric conductor; thus, for example, very little if any radio frequency current will be induced in a pair of rubber covered copper wires enclosed in an iron conduit, where the iron conduit is grounded at intervals.

Low potential circuits have often been protected from radio frequency potentials by grounding the low potential circuits thru high resistance rods made up of carbon and clay; and in some cases by using incandescent lamps between the conductors and ground. The writer has always considered this an objectionable practice, because to some extent it grounds the low potential circuits, which are usually better ungrounded. Also, according to the experience of the writer, these high resistance grounds have apparently offered, as a rule, greater impedance

to the radio frequency currents than small condensers offered. Slate switchboards sometimes served as protectors to low frequency circuits because their resistance was sufficiently low to allow them to act much as the high resistance protective rods.

Less trouble has occurred since more metal has been used in the construction of ships, in the form of bulkheads, decks, and supports. In addition, the doing away with wiring in wooden moulding and the substitution of metal moulding, conduit, and metal covered cable has prevented radio frequency currents from being produced in the direct current and audio frequency wiring. Lead covered wire has been used sometimes, but has occasionally caused trouble when the lead has been mechanically forced thru the insulation and against the copper. It is probably preferable to use lead covered wire in protecting metal conduit with drains in the lower portions of the conduit to take care of sweating, etc.

Besides preventing sparking, another reason for thoroly grounding the stays and mast guys on vessels was the assumption that less energy is absorbed from the radio waves by thoroly grounding these stays than that which would probably be consumed in the resistance over leaking insulators.

The increasing knowledge and improving practice of professional radio engineers decreases the probability of damage. However, inexperienced persons instal transmitting and receiving stations from time to time, using such various types of apparatus as their circumstances and knowledge provide. Such stations as these are frequently erected in private houses, where sparks may occur on combustible material, and where telephone and lighting wires may not be protected by conduit or grounded metal covered wire, and where the antennas may be above or may parallel nearby telephone, fire alarm, lighting, and power wires. It might be useful to offer a set of rules to cover the various possibilities, but that would require very careful study, if these rules were drafted, to prevent imposing hardship on the young experimenter and radio student, who generally is limited as to means.

Rules might be prepared by a Joint Committee of the American Institute of Electrical Engineers and The Institute of Radio Engineers. These would make a useful addition to the Underwriters' Rules and improve engineering practice.

The radio laws which require low decrement and practically single waves to be radiated from transmitters, made for the purpose of preventing interference, may serve as a protection against radio transmitters causing damage. These laws, with

their resulting regulations, aid in eliminating the plain aerial type of transmitter whereby the antenna was raised to excessively high potentials, and because of the lower decrement, nearby circuits, unless their natural period is somewhat near that of the radio frequency, may not be excited to such an extent. High group frequency transmitters and especially transmitters of constant amplitude waves use lower voltage for equal power, which results in lower voltages being induced in nearby conductors. These types of transmitters are coming into general use and the constant amplitude wave transmitters may be the transmitters of the future. Therefore, the probability of damage should continue to decrease.

When the current flows in an antenna, magnetic and electrostatic fields are produced around that antenna; therefore all conducting materials in these fields are conductors in the dielectric of a condenser, and at the same time they are conductors which cut a magnetic field. Considering the antenna as one plate and the earth as the other plate, and the air between all parts of the antenna and the earth as the dielectric, all conductors within this air space will be at a different potential from both the antenna and the earth, while this condenser is being charged. As the antenna may be periodically charged to a high potential, the conductors in the dielectric may be periodically at a high potential with respect to earth, depending on their distance from the earth and from the antenna. If these conductors are at any time raised to a potential sufficiently high to break down the solid or air insulation between them and earth, they will discharge or spark to earth. Now, if these conductors are carrying another current such as direct current with a direct current potential difference to ground, then the direct current will as a rule, it may possibly be said, follow the spark, and establish an arc which may hold until the circuit is opened by some portion burning away whether that portion be a fuse, a wire line, or generator winding. In the same manner both terminal wires of a motor or generator may spark simultaneously to the armature core, and produce a short circuit. This may occur whether or not the motor or generator is grounded because the motor or generator usually occupies a relatively different position in the dielectric from that occupied by the line wires.

Where conductors are within sparking distance of the antenna a discharge may take place, altho the conductors may be insulated from ground and from the antenna, and this for the same reason that such conductors discharge to ground. For example,

antenna circuits frequently discharge to such small masses of metal as wood screws, altho the surrounding wood is a good insulator.

Conductors are usually so placed as to cut the magnetic lines emanating from the antenna or the closed circuit of the transmitter; and a potential difference between the ends or portions of a system of such conductors may result which will break down the insulation. If these ends or portions are, for example, the opposite terminals of a motor or generator, or the terminals of a magnet, a short circuit may result. The electrostatic and magnetic fields may work together to produce such damage.

The shorter waves formerly used may have corresponded more nearly to the natural wave lengths of conductors which were found on shipboard in the lower potential circuits than do the longer waves used at present. When the conductor which tends to spark to ground or to the frame of a dynamo is connected to ground or to the frame thru a condenser, and where that condenser is relatively of much higher capacity than the capacity of a conductor to ground or to the object to which it tends to spark, the effect is probably somewhat similar to bringing the conductor quite near to the ground or the frame, and the nearer the conductor is to ground or to the frame, the lower the potential difference that exists between the conductor and ground or frame. That is, the conductor will be brought to a point of lower potential in the potential gradient between the antenna and ground. Or this protective condenser may be possibly regarded with fair correctness as a very low impedance so far as the radio frequency potential is concerned; and where a relatively low impedance is in circuit, the potential across that impedance must be relatively low. In other words, a relatively low impedance is provided for the radio frequency current around the insulation provided for the direct or audio frequency current.

Test made at the United Wireless Telegraph Company's Manhattan Beach station on Aug. 21, 1909, are interesting in these connections.

Referring to the Figure:

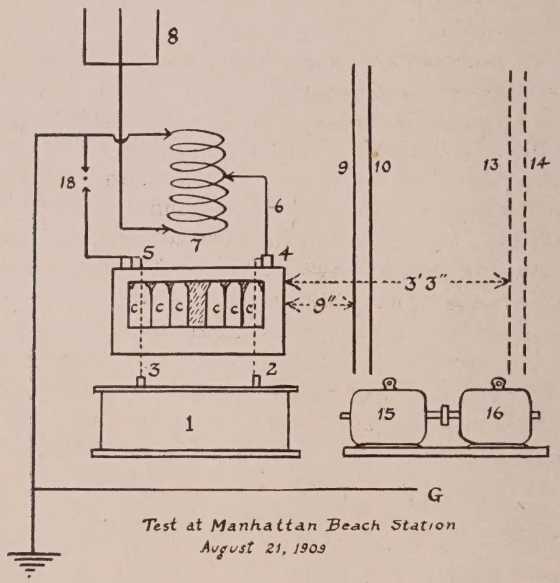
1, 2, 3, 4, 5, 6, 7, 8, 18, 19 represent the United Wireless transmitter.

1. 2 K. W. 60 cycles transformer.

2, 3, 4, 5. Connections between transformer secondary and condenser.

6. Connection to coupler.
7. Helical coupler (oscillation transformer).
8. Antenna.
18. Spark gap.
19. Ground.
c—*c* Condenser (Leyden jars).

9, 10, 13, 14 represent the relative position of two test wires each of number 10 B. & S. rubber covered copper wire* 5 feet (1.5 m.) in length. The wires were parallel and 3 inches (7.62 cm.) apart. These wires were open and insulated, and their lower ends were either brought near together or near to the earth or motor generator, so as to ascertain over what length of gap they would discharge.



15–16 represents a 2 K. W. Holtzer Cabot motor generator, on an iron base, insulated from ground. Motor (15), Generator (16). The motor generator was disconnected, power for the transmitter being taken from the city mains.

G. Wire connected to copper plate in earth, approximately 15 feet (5 m.) long.

*Diameter of number 10 wire = 0.102 inch = 0.259 cm.

The transmitter operated at full power, and radiated two waves (at 500 and 960 meters).

a. Test conductors at 9 and 10. No noticeable discharge between two lower ends.

b. Same at 13 and 14.

c. 9 or 10 discharges over 0.25 inch (0.6 cm.) air gap to ground (*G*).

d. 13 or 14 discharges over 0.17 inch (0.4 cm.) air gap to *G*.

e. 9 or 10 discharges over 0.06 inch (0.15 cm.) gap to motor generator (insulated from ground).

f. 13 or 14 discharges over 0.03 inch (0.08 cm.) gap to motor generator (insulated from ground).

g. 9 in grounded Greenfield conduit discharges approximately 0.01 inch (0.025 cm.) to ground (*G*).

h. 9 in ungrounded Greenfield discharges 0.22 inch (0.55 cm.) to *G*.

i. 9 in grounded Greenfield discharges 0.14 inch (0.35 cm.) to ungrounded motor generator.

j. 9 in Greenfield connected to motor generator, discharges less than 0.01 inch (0.025 cm.), to motor generator.

k. 9 in Greenfield connected to motor generator shows no discharge to motor windings.

l. 9 in grounded Greenfield discharges 0.14 inch (0.35 cm.) to motor windings.

m. 9 connected thru 0.013 μf. condenser to motor generator shows no discharge to either motor generator frame or motor windings.

n. Motor generator discharges 0.22 inch (0.55 cm.) to ground (*G*) when 9 is connected to motor generator thru 0.013 μf.

o. Motor generator shows no discharge to ground when connected to ground thru 0.015 μf. (conductor 9 was connected to motor generator thru 0.013 μf.).

Considering the antenna and earth together with the intervening air as a condenser, if we wish to protect conductors in this air dielectric against discharges from one plate or the other of this condenser, we must do one of two things: Either thoroly insulate the conductors to be protected or connect them electrically to the plate to which they have a tendency to discharge. That is, they must be thoroly insulated or made part of one plate or the other. As the insulation between low voltage circuits and ground is as a rule only sufficient to insulate the normal potential on the low voltage circuits, it is necessary to provide means for connecting these circuits to ground so far as radio

frequency currents are concerned, or to enclose them within the ground plate of the condenser rather than in dielectric.

The case is one of conductors subjected to alternating stress in the dielectric of a condenser and at the same time to an alternating magnetic field.

The problem is to prevent these conductors from sparking. The usual solution is to ground thoroly all conductors which are not there for the purpose of carrying current, and to enclose current carrying conductors in grounded metal coverings (e. g. metal conduit). Where this is not practicable, it is desirable to connect the current-carrying conductors to ground and to each other thru condensers (e. g. lead foil and mica condensers of approximately 0.17 μf. capacity tested at 500 volts, 60 cycle alternating current and enclosed in copper water-tight cases). In building a radio station, it is desirable to place all current-carrying conductors (other than radio) underground so far as practicable (and especially telephone conductors). The first continuous grounded metal deck of a vessel, below the radio transmitter, may be usually considered as the surface of the ground in so far as this protective effect is concerned.

SUMMARY: Various types of possible danger from radio installations are considered: shocks to the operator, short-circuit from lightning or from contact with high tension power lines, breakdown from the high tension circuits of the radio transmitter itself, and harmful inductive effects from radio transmitters.

In each case, instances are given together with the proper means of avoiding the undesired effect. In this connection, some detailed experiments are described.

DISCUSSION

Benjamin Liebowitz: In the protection of radio frequency apparatus, one of the most important points is the insertion of choke coils to localize properly the radio-frequency energy. I do not think it is as fully appreciated as it should be that multiple-layer coils are almost useless for this purpose. Because of their large effective distributed capacity, radio frequency currents are propagated with great ease thru such coils, and often with disastrous results. Thus, in one instance, I employed as a choke coil an inductance of about 600 turns of number 18 B. and S. wire* wound in 30 turns per layer, and burnt out a generator in consequence. I replaced this coil by six single-layer spirals about twenty-four inches (61 cm.) in inside diameter, each spiral having eighty turns of copper ribbon 0.50 by 0.01 inch (1.27 by 0.025 cm.) in section insulated by paper ribbon of the same section. The six spirals in series had somewhat less inductance than the multiple-layer coil first used, but to currents less than 100,000 cycles in frequency they were an almost perfect barrier. It cannot be too strongly emphasized that distributed capacity is just as undesirable in choke-coils as it is in radio frequency circuits.

* Diameter of number 18 wire = 0.040 inch = 0.010 cm.

SUSTAINED WAVE TRANSMISSION CHART*

By

Tyng M. Libby

(Bremerton Navy Yard)

Radio engineers are frequently called upon to estimate the range of radio transmitters, and to predetermine the height of antennas, wave length, and power required to cover a given range under average conditions. While estimates may be made from a wide experience with a large number of stations, the writer is of the opinion that closer approximations may be made by the calculation of these factors by means of semi-empirical formulae derived thru the correlation of data obtained thru tests. In order to determine which formula most nearly represents the observed results of daylight transmission, a comparison of the following sustained wave formulas has been made:

The Sommerfeld[1] theoretical formula:

$$I_r = 377 \frac{h_1 h_2 I_s}{\lambda d R} \sqrt{\frac{\theta}{\sin \theta}} \cdot \varepsilon^{-\frac{0.0019 d}{\sqrt[3]{\lambda}}} \tag{1}$$

The Austin[2] semi-empirical formula:

$$I_r = 377 \frac{h_1 h_2 I_s}{\lambda d R} \sqrt{\frac{\theta}{\sin \theta}} \cdot \varepsilon^{-\frac{0.0015 d}{\sqrt{\lambda}}} \tag{2}$$

The Fuller[3] semi-empirical formula:

$$I_r = 377 \frac{h_1 h_2 I_s}{\lambda d R} \sqrt{\frac{\theta}{\sin \theta}} \cdot \varepsilon^{-\frac{0.0045 d}{\lambda \sqrt{2}}} \tag{3}$$

The formula[4] given in Eccles' "Hand Book":

$$I_r = 4.25 \frac{h_1 h_2 I_s}{\lambda d} \cdot \varepsilon^{-\frac{0.0045 d}{\lambda^{\frac{3}{2}}}} \tag{4}$$

In these formulae, I_s and I_r are sending and receiving

*Presented before The Institute of Radio Engineers, Seattle Section, June 10, 1916.

[1] A. Sommerfeld, "Ann. der Phys.," 28, 1909.

[2] "Bulletin of the Bureau of Standards," volume 11, number 1, Nov., 1914.

[3] L. F. Fuller, "Proc. A. I. E. E.," volume 34, number 4.

[4] "Hand Book of Wireless Telegraphy and Telephony," Eccles.

antenna current respectively in amperes; h_1 and h_2 the effective heights in kilometers of the transmitting and receiving antennas, respectively; λ is the wave length and d the distance, both in kilometers; R is the radio frequency resistance in ohms, of the receiving system. The term $\sqrt{\frac{\theta}{\sin\theta}}$ accounts for the effect of the curvature of the earth, θ representing the angle, at the center of the earth, subtending the distance d. For practical purposes this term may be considered equal to unity.

Table 1 gives some of the results of receiving tests at Darien,[5] with audibilities calculated from formulas (1), (2), (3), and (4). The received watts corresponding to unit audibility[5] were taken as 1.23×10^{-15} and the audibility taken as proportional to the received current.

The values given by equation (1) are in fair agreement with the observed results at the shorter distances. For the longer distances, the calculated values are so low as to support the conclusion of Dr. Austin[5], that equation (1) represents the very lowest values of received energy, and that at the greater distances, these are strengthened by reflection from the upper strata.

Equation (3) gives absurdly high values as compared with the observed results, and might possibly be due to difference in types, and methods of manipulation of receiving apparatus, etc., when this formula was derived.

Equation (4) is in closer agreement with the observed values on shorter distances than the Austin formula. At the greater distances, however, the values are extremely high. This formula as given by Eccles, assumes a value of 25 ohms for R. In these computations, the term R was introduced into the equation, which changed the coefficient 4.25 to 106.25.

Attention should be called to the publication in different places of equations (3) and (4) as Fuller's equation. These two equations are not at all in agreement, and without a copy of Mr. Fuller's original paper,[3] one would be at a loss as to which was the "Fuller Formula."

Of the four equations, (2) gives the most consistent values, and as a whole may be taken as a close approximation. It is noticed that the audibility of Arlington as calculated by equation (2) is in good agreement with the observed value. It is suggested that this may be due to the fact that equation (2) was derived from data taken at that station.

[5] L. W. Austin, "Experiments at the U. S. Naval Radio Station, Darien, Canal Zone," "Proc. I. R. E.," volume 4, number 3, 1916.

TABLE I

Transmitting Station	I_s Amps.	λ Meters	h_1 Meters	R Ohms	Dist. Km.	Audibility				
						Obs.	Equa. 1	Equa. 2	Equa. 3	Equa. 4
Arlington	60	6000	61*	23.2	3330	5000	1840	7780	18180	6080
Tuckerton	115	7400	120	25	3430	10000	6040	25780	68680	22360
Sayville	140	9400	80	14	3520	7500	6120	25900	74650	18780
San Diego	35	3800	55	26.5	4670	0–100	113	911	1400	547
San Francisco	40	6500	96	23.5	4820	0–1000	295	2334	8540	2020
Honolulu	60	10000	96	13.5	8500	150	16	514	6640	2440
Nauen	150	9400	120	29	9400	200	16	750	1260	4860
Eilvese	140	7400	120	25	9160	200	10	500	7030	2840

* h_1 corrected from short range observations and found to be approximately one-half of height to geometric center. h_1 for other stations taken as 80 per cent. of height to geometric center.

Effective height of Darien antenna, 146 meters.

The writer has made many audibility tests on naval ships and shore stations, and while the data obtained cannot be disclosed, it may be stated that the values calculated by equation (2) were found to be in fair agreement with the observed values.

A simple algebraic solution of equation (2) for the values λ and d is impossible, since they occur linearly and exponentially. For the purpose of comparing observed results with equation (2) predetermining λ, h_1, h_2 and I_s to cover a given range, and for readily solving for d, a chart has been prepared by the writer. This chart is similar to the one for spark transmitters, which was submitted for publication by the writer's co-worker, Mr. H. G. Cordes.[6]

The radio frequency resistance of the average receiving system is about 25 ohms. Assuming this value for R and stating equation (2) in English units:—

$$I_r = 757 \frac{h_1 h_2 I_s}{\lambda d} \cdot \varepsilon^{-\frac{0.0877 d}{\sqrt{\lambda}}} \qquad (5)$$

I_r in micro amperes.
I_s in amperes.
h_1 and h_2 in feet.
λ in meters.
d in nautical miles.

Dividing (5) algebraically, and stating logarithmically:

$$log_{10} \frac{I_r}{757} + log_{10} \frac{\lambda}{h_1 h_2 I_s} = \frac{0.0877 d}{\sqrt{\lambda}} log_{10} \varepsilon - log_{10} d \qquad (6)$$

A number of curves (the broken curves in the chart) for various values of I_r are plotted with ordinates $\frac{\lambda}{h_1 h_2 I_s}$, and abscissas,

$$log \frac{I_r}{757} + log \frac{\lambda}{h_1 h_2 I_s}$$

For each of a number of wave lengths additional curves (full line curves in the chart) are plotted over the same abscissa equated thru the expression

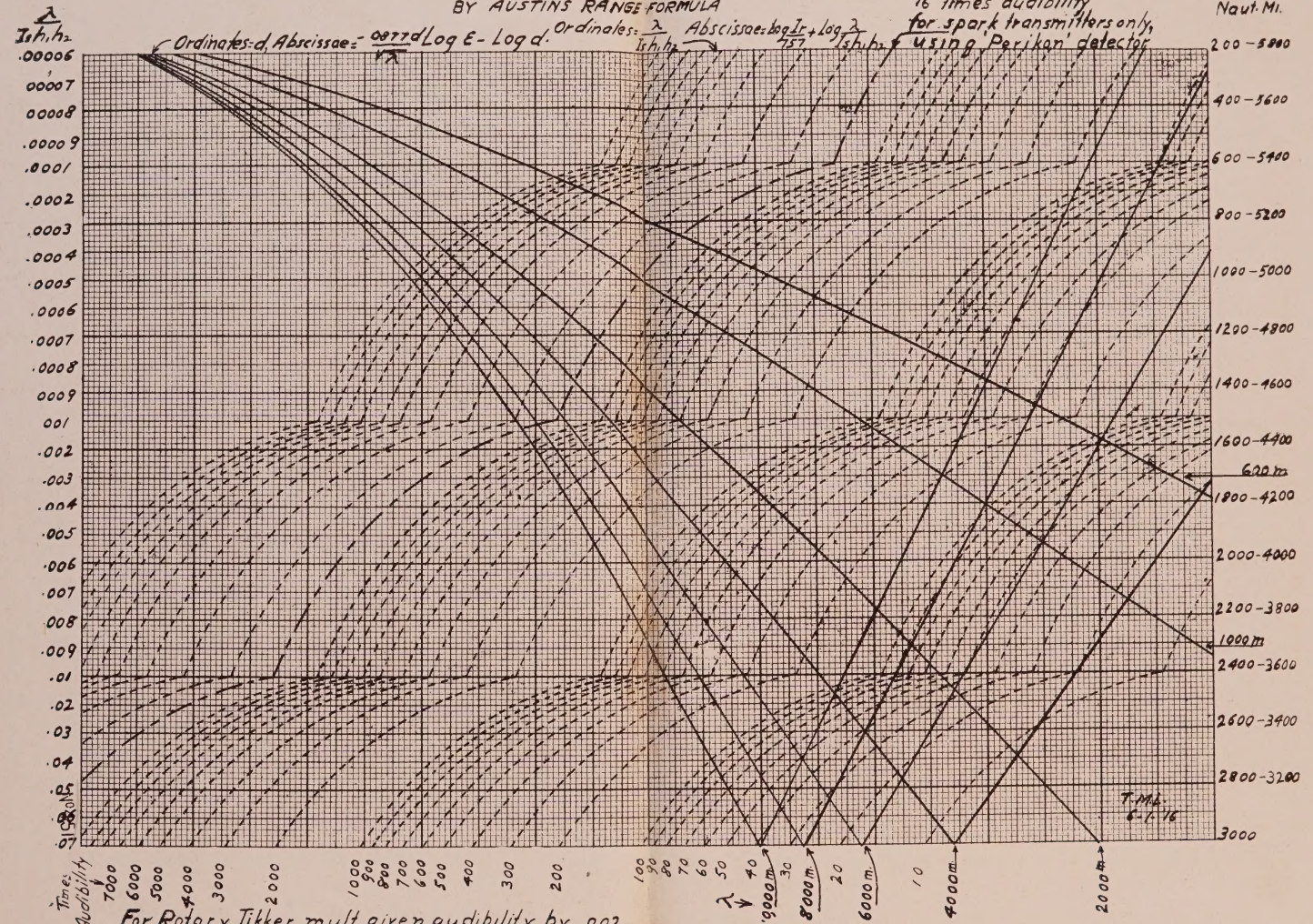
$$-\frac{0.0877 d}{\sqrt{\lambda}} log \varepsilon - log d$$

with d as ordinates.

The I_s curves (broken line) are marked in terms of audibility using the oscillating audion as a detector, the received energy required for unit audibility being taken as 1.23×10^{-15} watts[5],

[6] "Electrical World," volume 66, number 23, 1915.

APPROXIMATE DAYLIGHT TRANSMISSION RANGE OVER SEA WATER
BY AUSTIN'S RANGE FORMULA



For Rotary Tikker mult. given audibility by .002.

and the audibility assumed to be proportional to I_r. The current required for unit audibility thru 25 ohms, using the ultraudion detector, is therefore 0.007×10^{-6} amperes.

Fuller found that the received energy required for unit audibility using a rotary tikker is 3.2×10^{-10} watts[3], or 3.56×10^{-6} amperes thru 25 ohms. Since the audibility with the tikker detector varies directly as the current, the audibility curves in the chart may be used for that detector by multiplying the given audibility by $\frac{0.007 \times 10^{-6}}{3.58 \times 10^{-6}}$ or 0.002.

If using a detector other than the ultraudion, the value of the curves in terms of I_r can readily be determined from their values expressed in audibility.

To solve for d with a given set of transmitter values, proceed as follows:

1. Compute $\frac{\lambda}{h_1 h_2 I_s}$.

2. Locate this value on the left-hand ordinate scale and follow in horizontally to the intersection of the broken curve for the audibility desired.

3. From this intersection, proceed vertically to the solid line for the wave length used.

4. From this last intersection, proceed horizontally to the right, and read the required value of d on the right-hand ordinate scale.

To predetermine the value of $\frac{\lambda}{h_1 h_2 I_s}$ required to cover a given range, the operations are just the reverse of these for determining d.

To find the value of I_r as calculated by equation (2) when all other factors are known:

1. Locate d on the right-hand ordinate scale, and follow in horizontally to the solid curve for the wave length used. (Note the abscissa at this intersection.)

2. Compute $\frac{\lambda}{h_1 h_2 I_s}$.

3. Locate this value on the left-hand ordinate scale, and follow in horizontally to the abscissa noted in operation 1. The audibility curve upon which this last intersection lies, is the audibility required. If this last intersection does not lie exactly on one of the audibility curves plotted, it is but a simple matter to interpolate. To express the audibility in micro-amperes thru 25 ohms, multiply by 0.007.

In predeterminations, under ordinary conditions, the writer uses a factor of safety of 12 times the minimum audibility required. In exceptional cases, such as high intervening mountains, in short distances, and in the use of short wave lengths, a factor of safety of 16 times the audibility required, would not be too large.

The Austin-Cohen[7] daylight transmission formula for damped transmitters

$$I_r = 4.25 \frac{h_1 h_2 I_s}{\lambda d} \cdot \varepsilon^{-\frac{0.0015 d}{\sqrt{\lambda}}} \qquad (7)$$

has been found to agree fairly well with observed results, on wave lengths up to 4000 meters and distances up to 2000 miles, where the transmitters were coupled loosely enough to radiate but one wave.

Expressed in English units (7) becomes

$$I_r = 212 \frac{h_1 h_2 I_s}{\lambda d} \cdot \varepsilon^{-\frac{0.0877 d}{\sqrt{\lambda}}}$$

Transposing and stating logarithmically

$$log_{10} \frac{I_r}{212} + log_{10} \frac{\lambda}{h_1 h_2 I_s} = -\frac{0.0877 d}{\sqrt{\lambda}} log_{10} \varepsilon - log_{10} d \qquad (9)$$

It will be noticed that the right side of equation (9) is identical with that of equation (6). The full line curves for wave lengths in the chart, therefore, are the same for damped and sustained wave transmitters.

If in the chart, a set of I_r curves were plotted with ordinates $\frac{\lambda}{h_1 h_2 I_s}$ and abscissas $log \frac{I_r}{212} + log \frac{\lambda}{h_1 h_2 I_s}$, this chart could be used for both damped and sustained wave transmitters.

Dr. Austin has found that an audibility of sixteen, or 28×10^{-6} amperes thru 25 ohms, insures good communication thru strays and interference, using the electrolytic, or the perikon detectors.

In order to make the chart applicable to damped transmission, the dot and dash curve has been plotted, the value of I_r being taken as 28×10^{-6} amperes.

While it is admitted that a transmission theory, rather than a transmission formula is desired, I do not think that more sustained wave transmission data would be undesirable, and I make this attempt to stimulate activity in this line.

[7] "Bulletin, Bureau of Standards," volume 7, number 3, 1911; volume 11, number 1, 1914.

SUMMARY: The Sommerfeld, Austin, Fuller, and Eccles transmission formulas are compared with available data, and the conclusion reached that the Austin formula is most nearly correct. A chart is given whereby, given any five of the six quantities: wave length, transmitting and receiving antenna heights, distance of transmission, transmitting current, and received audibility, the sixth can readily be obtained. Tho specially intended for sustained wave reception using the ultraudion, it is shown that the chart can also be used for other detectors and damped wave reception.

QUANTITATIVE RELATIONS IN DETECTOR CIRCUITS*

(A Discussion on Mr. Armstrong's Paper on "A Study of Heterodyne Amplification by the Electron Relay.")

By

Benjamin Liebowitz, Ph. D.

The object of this discussion is to bring out certain fundamental relations in simple detector circuits, and thereby to determine directly the maximum amplification which can be attributed to the heterodyne principle. The chief assumptions are (1), that the predominant reaction in the detector circuit (D K of Figure 1) is that due to the resistance of the detector D, and (2), that this resistance is so high that the energy abstracted by the detector circuit from the oscillatory circuit LC is very small. Under these circumstances, there will be an

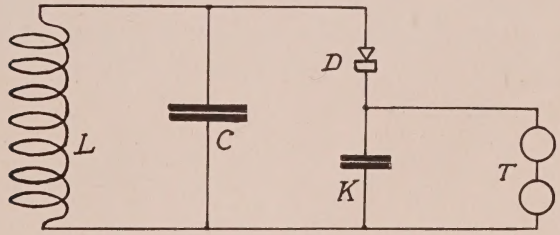


Figure 1—Typical Detector Circuit

approximately simple harmonic e.m.f. impressed on the detector if the received signals are simple harmonic, as we shall suppose, and the current thru the detector can then be determined.

Two types of detectors will be considered, viz., "perfect" rectifiers and "approximate" rectifiers. The method of procedure in each case is to compute the radio frequency currents as well as the audio frequency currents flowing in the detector circuit, and by comparison of these to determine what part of the

* Received by the Editor, November 1, 1916.

energy abstracted from the oscillatory circuit is useful in producing an audible frequency telephone current.

Case I. "Perfect" Rectifiers

A "perfect" rectifier may be defined as one which has a constant resistance in one direction and an infinite resistance in the other direction. The voltage-current characteristic for such a detector is shown in Figure 2.

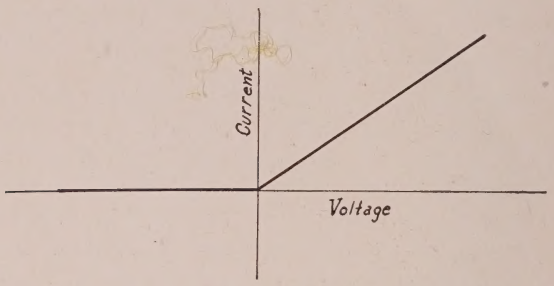


Figure 2—Characteristic of "Perfect" Rectifier

When a simple harmonic e.m.f. is impressed on a rectifier of this kind, the resulting current will obviously be a succession of sine loops, such as shown in Figure 3. If the e.m.f. is $e \sin pt$

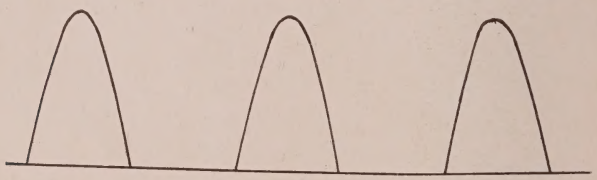


Figure 3—Current Thru "Perfect" Rectifier

and the finite resistance of the detector is R, the amplitude of these loops will be simply $\frac{e}{R}$.

This succession of loops is readily decomposable into a Fourier's Series. By the usual process the series is found to be

$$\frac{e}{R}\left\{\frac{1}{\pi}-\frac{1}{2}\cos pt+\frac{2}{3\pi}\cos 2pt-\frac{2}{15\pi}\cos 4pt+\cdots\right\}$$

We have here a rectified current of magnitude $\frac{1}{\pi}\frac{e}{R}$, a fundamental radio frequency current of amplitude $\frac{1}{2}\frac{e}{R}$, and a series of smaller overtones. If the rectified current is denoted by y_o and the amplitude of the fundamental radio frequency current by Y_p, it follows, therefore, that

$$y_o = \frac{2}{3} Y_p, \text{ approximately.}$$

Hence we see that *the magnitude of the useful current is only about two-thirds of the amplitude of the fundamental radio frequency current flowing in the detector circuit.*

The average rate at which energy is being drawn from the oscillatory circuit $L\,C$ is $\frac{1}{4}\cdot\frac{e^2}{R}$ (being half of that which would obtain if the detector had resistance R in both directions). Of this energy, $\frac{1}{\pi^2}\cdot\frac{e^2}{R}$ is in rectified current, $\frac{1}{8}\cdot\frac{e^2}{R}$ is in fundamental radio frequency current, and the rest is in the overtones. That is to say, *of the total energy abstracted from the oscillatory circuit, about 40 per cent. is in the form of rectified current, 50 per cent. in the form of radio frequency current of fundamental period, and the remainder is in the overtones.*

In order to be heard, the continuous series of loops of Figure 3 may be broken up into trains of audible frequency, so as to convert the steady rectified current into one which rises and falls between zero and $\frac{1}{\pi}\cdot\frac{e}{R}$. This is equivalent to an audio frequency current of amplitude $\frac{1}{2\pi}\cdot\frac{e}{R}$, superimposed on a direct current, plus overtones. The energy relations are otherwise unchanged, for the preceding analysis is applicable to each train or to each loop of each train.

Instead of being broken up, however, the series of loops may be made audible by the heterodyne method. If the "other force" is of the same magnitude as the incoming e.m.f. (the "equal" heterodyne), then the loops will rise and fall between zero and $\frac{2e}{R}$. And since each loop is very nearly pure sine in shape, the above analysis is applicable as a close approximation, and we therefore obtain a beat frequency current of amplitude $\frac{1}{\pi}\cdot\frac{e}{R}$; which is twice that obtained without the heterodyne.

When, however, the local force, say of amplitude E, is large compared with that received, then the loops will rise and fall between $\frac{E+e}{R}$ and $\frac{E-e}{R}$. And if we applied the preceding analysis, we should still find that the amplitude of the beat frequency current is $\frac{1}{\pi} \cdot \frac{e}{R}$. So that *if the detector is a perfect rectifier, the heterodyne method gives twice as great a useful telephone current as the "breaking up" method, irrespective of the amplitude of the local current*; a result entirely in accord with that at which I arrived originally.[1]

In the sense that with a given "perfect" rectifier the heterodyne gives twice as great a telephone current as the "breaking up" method gives, and hence four times as much energy in the response, the heterodyne may be said to give a four-fold true amplification. This interpretation of "true amplification" was used by Mr. Armstrong in his paper, and is entirely justifiable; but it must not be taken to mean that the heterodyne puts four times as much energy into the telephone current as received frequency energy abstracted from the oscillatory circuit. This question will be taken up in greater detail in the next section.

The reason for the four-fold amplification becomes perfectly clear if we bear in mind that in the "breaking up" method of receiving sustained waves we subtract roughly half the available energy, whereas with the heterodyne method we add as much available energy as we had to begin with.

I have omitted many of the details of the preceding analysis and passed over several points, because, after the Fourier's series has been worked out for the succession of sine loops of Figure 3, the subsequent results become clear by physical reasoning, and also because the case of "approximate" rectifiers, which will now be taken up, is of much greater interest.

Case II. "Approximate" Rectifiers

The characteristic of an ordinary crystal rectifier differs from that of a "perfect" rectifier (shown in Figure 2) in that it runs slightly below instead of along the current axis for negative voltages, in that it has a finite instead of an infinite curvature at the origin, and in that it generally curves upward instead of being straight for positive voltages. The ordinary rectifier is therefore imperfect in two respects: it rectifies no alternating

[1] See these Proceedings, June, 1915, page 185, *et seq.*

current completely, and it rectifies relatively large currents better than small.

For purposes of analysis it is desirable to use a detector which rectifies small currents substantially as well as relatively large ones. To do this, the characteristic of the detector must have a rapid, tho finite, curvature at the origin, as shown in Figure 4. Such a detector will be assumed in our analysis, and will be called an "approximate rectifier."

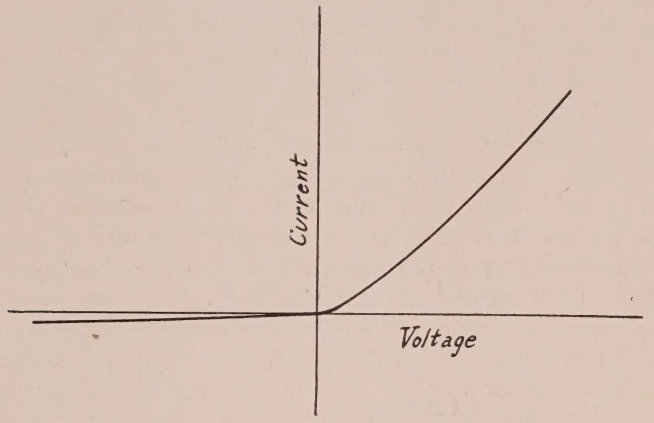


FIGURE 4—Characteristic of "Approximate" Rectifier

The curve of Figure 4 can be represented by a power series, or, with sufficient accuracy for all practical purposes, by a dozen or so terms of such a series. Furthermore, in order to satisfy our definition of an "aproximate rectifier," the sum of all the odd powers of the series must be nearly equal to the sum of all the even powers; for the odd powers change sign when the voltage becomes negative, whereas the even powers do not; so that if their sums are nearly equal, they will nearly cancel out for negative voltages and the series will be large only for positive voltages. Hence, if y is the current and v the voltage, the characteristic of our approximate rectifier is represented by

$$y = a_1 v + a_2 v^2 + a_3 v^3 + a_4 v^4 + \cdots, \tag{1}$$

subject to the condition, when v is positive that

$$a_1 v + a_3 v^3 + a_5 v^5 + \cdots = a_2 v^2 + a_4 v^4 + a_6 v^6 + \cdots + \Delta, \tag{2}$$

where Δ is a small quantity depending on v.

In order to deal with the simplest and most favorable case, moreover, the coefficients a_k will all be taken as positive; i. e., the detector characteristic will be assumed to be one in which, within the given range, the current increases more and more rapidly with increasing positive voltage.

Suppose, now, that, due to the received signals acting alone, there is a simple harmonic e.m.f. $e \sin p t$ impressed on the detector. The resulting current will be:

$$y = a_1 e \sin p t + a_2 e^2 \sin^2 p t + a_3 e^3 \sin^3 p t + \cdots \quad (3)$$

Remembering that

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin^3 \theta = \frac{1}{4}(3 \sin \theta - \sin 3\theta)$$

$$\sin^4 \theta = \frac{1}{8}(3 - 4 \cos 2\theta + \cos 4\theta)$$

etc.,

we get:

$$\begin{aligned} y = {} & a_1 e \sin p t + \frac{1}{2} a_2 e^2 (1 - \cos 2 p t) \\ & + \frac{1}{4} a_3 e^3 (3 \sin p t - \sin 3 p t) \\ & + \frac{1}{8} a_4 e^4 (3 - 4 \cos 2 p t + \cos 4 p t) \\ & + \frac{1}{16} a_5 e^5 (10 \sin p t - 5 \sin 3 p t + \sin 5 p t) \\ & + \frac{1}{32} a_6 e^6 (10 - 15 \cos 2 p t + 6 \cos 4 p t - \cos 6 p t) \\ & + \cdots \cdots \cdots \cdots \cdots \\ & + \cdots \cdots \cdots \cdots \cdots \end{aligned} \quad (4)$$

Grouping the terms of this series according to periodicity, and denoting the current of zero frequency by y_o, that of frequency $\frac{p}{2\pi}$ by y_p, we find:

$$y_o = \frac{1}{2}\left\{ a_2 e^2 + \frac{3}{4} a_4 e^4 + \frac{3 \times 5}{4 \times 6} a_6 e^6 + \frac{3 \times 5 \times 7}{4 \times 6 \times 8} a_8 e^8 + \cdots \right\} \quad (5)$$

$$y_p = \sin p t \left\{ a_1 e + \frac{3}{4} a_3 e^3 + \frac{3 \times 5}{4 \times 6} a_5 e^5 + \cdots \right\} \quad (6)$$

Equations (5) and (6) are general formulas for calculating the rectified and radio frequency currents flowing thru a simple detector circuit of known characteristic. It should be

noted that these formulas do not depend on any of the assumptions we have made regarding the shape of the detector characteristic.

Suppose now, to fix the ideas, that ten terms of our series (1) are sufficient to represent the detector characteristic. The last term of the bracketed expression in (5) will then be

$$\frac{3\times5\times7\times9}{4\times6\times8\times10}\,a_{10}\,e^{10}=0.492\,a_{10}\,e^{10};$$

and the last term of the bracket in (6) will be $0.492\,a_9\,e^9$. Since the coefficients a_k are positive, we get, therefore, the following inequalities:

$$a_2\,e^2+a_4\,e^4+\cdots+a_{10}\,e^{10}>a_2\,e^2+\frac{3}{4}a_4\,e^4+\cdots+0.492\,a_{10}\,e^{10}$$
$$>0.492\,(a_2\,e^2+a_4\,e^4+\cdots+a_{10}\,e^{10}),$$

$$a_1\,e+a_3\,e^3+\cdots+a_9\,e^9>a_1\,e+\frac{3}{4}a_3\,e^3+\cdots+0.492\,a_9\,e^9$$
$$>0.492\,(a_1\,e+a_3\,e^3+\cdots+a_9\,e^9).$$

Introducing now our assumption regarding the shape of the characteristic, i. e., the relation (2), it follows that the brackets in (5) and (6) must be of the same order of magnitude. Hence, indicating the amplitude of y_p by Y_p, we may write, as a rough approximation:

$$y_o=\frac{1}{2}\,Y_p\ \text{(roughly)} \qquad (7)$$

For the case of the "perfect" rectifier, on the other hand, we found $y_o=\frac{2}{3}\,Y_p$ approximately.

Thus, the widely different methods of analysis give results of the same order of magnitude, and since, from our conception of an "approximate" rectifier, we should except this to be the case from physical reasoning, the comparison affords an excellent check on the mathematical deductions

Turning now to the behavior of the heterodyne method when used in conjunction with an "approximate" rectifier, let there be a local e.m.f. $E\sin qt$ impressed on the detector, and let this be large in comparison with the received e.m.f. $e\sin pt$. The voltage v in series (1) now becomes $E\sin qt+e\sin pt$. In the binomial expansion of the expressions $(E\sin qt+e\sin pt)^n$, two terms give a sufficiently close approximation, since $\frac{e}{E}$ is as-

sumed small. There results, therefore, for the current in this case:

$$\begin{aligned} y = {} & a_1 (E \sin q t + e \sin p t) \\ & + a_2 (E^2 \sin q t + 2 E e \sin p t \sin q t) \\ & + a_3 (E^3 \sin^3 q t + 3 E^2 e \sin^2 q t \sin p t) \\ & + a_4 (E^4 \sin^4 q t + 4 E^3 e \sin^3 q t \sin p t) \\ & + \cdots \\ & + \cdots \end{aligned} \tag{8}$$

Expanding the terms $\sin^k q t$ and $\sin^{k-1} q t \sin p t$ by well known or easily derived formulas into polynomials involving multiple angles, we get:

$$\begin{aligned} y = {} & a_1 \{E \sin q t + e \sin p t\} \\ & + a_2 \left\{ \frac{1}{2} E^2 (1 - \cos 2 q t) - E e [\cos (q+p) t - \cos (q-p) t] \right\} \\ & + a_3 \left\{ \frac{1}{4} E^3 (3 \sin q t - \sin 3 q t) \right. \\ & \qquad \left. + \frac{3}{4} E^2 e [\sin (2q-p) t - \sin (2q+p) t + 2 \sin p t] \right\} \\ & + a_4 \left\{ E^4 \left(\frac{3}{8} - \frac{1}{2} \cos 2 q t + \frac{1}{8} \cos 4 q t \right) \right. \\ & \qquad + E^3 e \left[\frac{3}{2} \cos (p-q) t - \frac{3}{2} \cos (p+q) t \right. \\ & \qquad \left. \left. - \frac{1}{2} \cos (3q-p) t + \frac{1}{2} \cos (3q+p) t \right] \right\} \\ & + \cdots \\ & + \cdots \end{aligned} \tag{9}$$

Grouping all the terms of frequency $\frac{1}{2\pi}(p-q)$ under the head y_{p-q}, and those of frequency $\frac{1}{2\pi} p$ under the head y_p, there results:

$$y_{p-q} = \left\{ a_2 E + \frac{3}{2} a_4 E^3 + \frac{3 \times 5}{2 \times 4} a_6 E^5 + \frac{3 \times 5 \times 7}{2 \times 4 \times 6} a_8 e^7 + \cdots \right\} e \cos (p-q) t \tag{10}$$

$$y_p = \left\{ a_1 + \frac{3}{2} a_3 E^2 + \frac{3 \times 5}{2 \times 4} a_5 E^4 + \frac{3 \times 5 \times 7}{2 \times 4 \times 6} a_7 E^6 + \cdots \right\} e \sin p t \tag{11}$$

Equations (10) and (11) are independent of any of the assumptions we have made regarding the shape of the detector

characteristic, and are, therefore, general formulas for calculating the beat frequency and received radio frequency currents flowing thru a known heterodyne detector circuit, when the local E. M. F. is large in comparison with that receiver.

Again supposing that ten terms of the series are sufficient, the last term of the bracket in (10) becomes $2.46\, a_{10} E^9$, and of that in (11), $2.46\, a_9 E^8$. Hence there results the inequalities:

$$a_2E+a_4E^3+ \cdots +a_{10}E^9 < a_2E+\frac{3}{2}a_4E^3+ \cdots +2.46\,a_{10}E^9$$
$$< 2.46\,(a_2E+a_4E^3+ \cdots +a_{10}E^9), \quad (12)$$

$$a_1+a_3E^2+ \cdots +a_9E^8 < a_1+\frac{3}{2}a_3E^2+ \cdots +2.46\,a_9E^8$$
$$< 2.46\,(a_1+a_3E^2+ \cdots +a_9E^8). \quad (13)$$

Also, from (2) we get

$$a_1+a_3E^2+a_5E^4+ \cdots = a_2E+a_4E^3+a_6E^5+ \cdots +\frac{\Delta}{E}. \quad (14)$$

And since the slope of the curve of Figure 4 is small at the origin, $\frac{\Delta}{E}$ will be a small quantity and $a_1+a_3E^2+ \cdots$ will therefore be nearly equal to $a_2E+a_4E_3+ \cdots$. From (12), (13), and (14) it follows, therefore, that the brackets in (10) and (11) must be of the same order of magnitude. Hence, denoting the amplitude of y_p by Y_p and that of y_{p-q} by Y_{p-q}, we may write, as a rough approximation,

$$Y_{p-q} = Y_p \qquad \text{(roughly)}. \quad (15)$$

While we have used a relatively small number of terms of the series (1) to get the result (15), the truth thereof would not be affected even if it were necessary to use many more terms. And we may be sure that in all practical cases the rectifier characteristic can be quite accurately specified by comparatively few terms.

A comparison of (15) and (7) shows that for a given Y_p, i. e., for a given amount of received frequency energy in the detector circuit (which is the only fair basis of comparison), the heterodyne method gives roughly twice as much audible frequency current and hence four times as much energy in the response as the "breaking up" methods give, when the detector is an "approximate" rectifier. In this sense, the four-fold amplification of the heterodyne method has once more been demonstrated, but *only* in this sense. For, inasmuch as the received frequency energy abstracted from the oscillatory circuit must

be but a small fraction of the total received energy, the result (15) proves that *no matter how large the local heterodyne current may be, the energy in the response must always be less than the energy in the signal.*

The assumption in this analysis of the "approximate" rectifier characteristic of Figure 4 brings out the best that both methods are capable of. In general, however, practical rectifiers do not have as rapid a curvature at the origin as we have assumed here, i. e., they rectify large currents much better than small. Hence the departures of practical characteristics from condition (2) will lessen the effectiveness of the "breaking up" methods far more than that of the heterodyne method. It is for this reason that considerably more than a four-fold amplification is obtained by the heterodyne method, as was brought out very clearly by Mr. Armstrong in his paper. In short, the real advantage of the heterodyne method, aside from the production of a musical tone, lies in the more efficient use of the detector characteristic than is possible without it.

The results contained herein do not reflect any discredit whatever on the heterodyne principle itself, but only upon those theories which purport to show that more could be done therewith than the laws of nature would allow.

SUMMARY: In the case of a simple receiving system it is shown that even with a "perfect" rectifier, as defined, there is more high-frequency energy in the detector circuit than is associated with the rectified current. Also, with a "perfect" rectifier it is shown that the heterodyne method gives just four times the energy in the response as the "breaking up" methods give, irrespective of the amplitude of the local E. M. F.

"Approximate" rectifiers are defined. General formulas are derived for calculating the rectified current and the received radio frequency current flowing in a simple detector circuit of known characteristic; also, general formulas for calculating the beat frequency current and the received radio frequency current in the case of the heterodyne, when the local E. M. F. is larger compared with that received. From these formulas, and from the definition of an "approximate" rectifier, it is shown that the energy in the response must always be less than the energy in the signal.

NOTES ON A NEW METHOD FOR THE DETERMINATION OF THE MAGNETIC FLUX DENSITY AND PERMEABILITY*

By
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The following is an outline of a new method for determining the flux density, i. e., number of lines of magnetic induction per unit cross section, up to any desired frequency. The same arrangement may also be conveniently used for obtaining the magnetic permeability and for investigating the total loss of a coil containing a ferro-magnetic core, or the core and copper losses separately.

Principle and Theory of the Method—Explanation of the Arrangement

The suggested arrangement is shown in Figure 1 and is based on the application of a differential system, which has been recently described by the author.[1] One differential branch contains the test sample which has a definite coefficient of self-induction, L_x, for a particular current at a fixed frequency. The test sample is investigated by means of balancing its effect against a standard variable self induction, L_s, (variometer, air-core coils) in series with a non-inductive resistance r.

The performance of such a differential system is briefly as follows: When the currents in the two branches of the system are equal in effective value and in phase, their inductive effects on the secondary coil of the differential transformer will exactly neutralize each other, and no voltage will be induced in the coil. This is based on the assumption that the two primary coils, P_1 and P_2, are symmetrically placed with reference to the secondary coil, S, and have exactly the same number of turns which are wound in opposite directions. Any kind of alternating cur-

* Received by the Editor, October 1, 1916.
[1] A. Hund, "Electrical World," May 22, 1915; reprinted in "London Electrician," August 27, 1915.

rent detector connected across the terminals of the secondary coil will then give a no-current indication when the currents in P_1 and P_2 are equal and in phase.

A no-current adjustment is established when the coefficient of self-induction of the variometer is equal to the effective coefficient of self induction of the test coil and when the effective resistance of the test coil is exactly balanced by the resistance

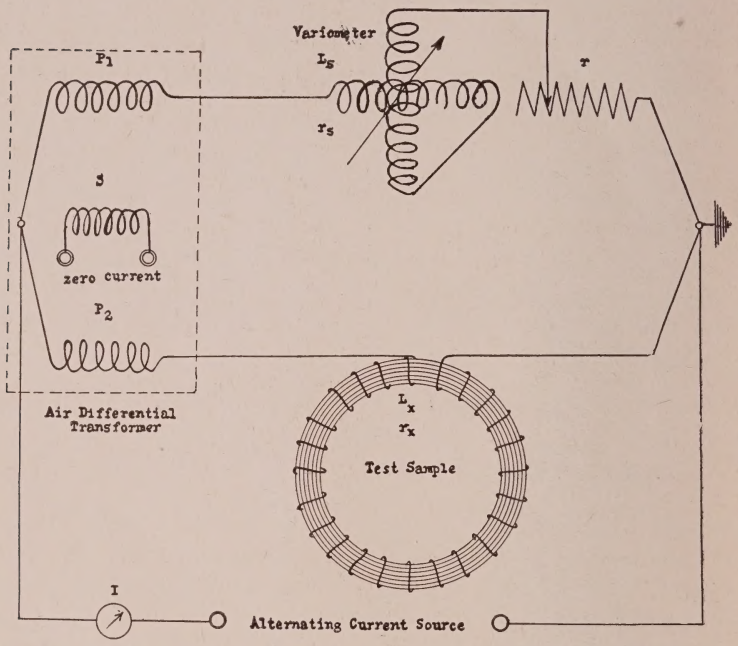

FIGURE 1

of the variometer and the series resistance r. An absolute disappearance of the differential field, however, can generally not be obtained since the wave form in the one branch is somewhat distorted due to the presence of the ferro-magnetic substance. In most practical cases, however, the minimum of the differential field can be very readily and accurately detected. For very precise measurements, it is advantageous to insert a condenser in series with the indicator in the secondary circuit of the differential transformer and tune this circuit to resonance with the required frequency. (This is especially recommended at higher

frequencies since a tuning is then readily obtainable.) For frequencies up to about 2,000 cycles, a Wien vibration galvanometer may be used as a current indicator, and the condenser may accordingly be dispensed with.

Derivation of the Expression of the Maximum Flux Density

When the ferro-magnetic core of a coil is exposed to an alternating flux of maximum value, Φ_{max}, the total change in the lines of induction, which go thru the cross sectional area of the iron core, during one half wave, is from zero to Φ_{max} and back to zero, i. e., a total change of $2\,\Phi_{max}$. When f denotes the number of cycles per second and T the corresponding, period, the average rate of change is $\frac{2\,\Phi_{max}}{\frac{T}{2}}$. Hence the average induced voltage per each turn of the coil is

$$E_{av} = 4\,f\,\Phi_{max}\,10^{-8} \text{ volts} \tag{1}$$

and the effective value for N turns is equal to

$$E = 4\,F\,f\,N\,\Phi_{max}\,10^{-8} \text{ volts} \tag{2}$$

where F denotes the form factor of the voltage wave, i. e., the ratio of effective value to average value. In case the flux traversing the core follows a sine law, the instantaneous value of it at any time, t, is defined as

$$\Phi_t = \Phi_{max}\,sin\,(2\,\pi\,f\,t)$$

The form factor F as determined according to the above definition is

$$F = \frac{\sqrt{\frac{2}{T}\int_o^{\frac{T}{2}} e^2\,d\,t}}{\frac{2}{T}\int_o^{\frac{T}{2}} e\,d\,t} = \frac{\frac{E_{max}}{\sqrt{2}}}{\frac{2\,E_{max}}{\pi}} = 1.111$$

and equation (2) becomes

$$E = 4.44\,f\,N\,\Phi_{max}\,10^{-8} \text{ volts} \tag{3}$$

When this expression is applied to the arrangement under discussion and the maximum flux density, B_{max}, is introduced, we find the expression for the induced E. M. F. of the test sample as

$$E = 4.44\,f\,N\,S\,B_{max}\,10^{-8} \text{ volts} \tag{4}$$

in which relation S stands for the cross sectional area of the

iron core. Now let the two inductances, L_x and L_s, be adjusted to the same value; i. e., first, adjustment of phase, and the resistance, r, regulated until the indicator of the differential transformer shows no effect whatever, and second, adjustment of amplitude. Then we may write[2]

$$4.44 f N S B_{max} 10^{-8} = 2\pi f L_s \frac{I}{2} \tag{5}$$

which leads to

$$B_{max} = 0.7075 \frac{L_{s\,(\text{henrys})}}{N S_{(\text{cm.})^2}} I_{(\text{amps.})} 10^8 \quad \begin{matrix}\text{lines of induction} \\ \text{per square centimeter}\end{matrix} \tag{6}$$

the expression for the maximum flux density in terms of L_s as read on the variometer; I, as measured by the ammeter in the main branch of the differential arrangement; N, the number of turns of the test sample; and, S, the cross sectional area of the iron core.

Determination of the Resultant Field Intensity H_{max} and Magnetic Permeability μ

One way of exploring the magnetic properties of a ferromagnetic substance by means of this method is to use a circular ring on which is uniformly wound a coil of wire of comparatively low resistance. Then the maximum magnetizing force, H_{max}, is defined by the formula[3]

$$H_{max} = \frac{4\pi}{10} \sqrt{2} \frac{N I_{m\,(\text{amps})}}{l_{\text{cm}}} \quad \begin{matrix}\text{gilberts per} \\ \text{centimeter}\end{matrix} \tag{7}$$

where I_m is the magnetizing component of the effective current traversing the coil of the sample, N the number of turns in the coil, and l the mean length of the magnetic path in centimeters. It is to be borne in mind that this equation holds only approximately when the diameter of the ring is large as compared with the diameter of the cross section. Suppose that in Figure 2 L_x and r_x denote the effective coefficient of self induction and apparent resistance of the sample and that they are exactly balanced by L_s and (r_s+r); that is, the self induction of the variometer, the resistance of it, and the additional balance resistance r. The vector diagram of Figure 3 then shows the voltage relations in the test coil and objects used for comparison. In this diagram V denotes the effective terminal pressure of the test sample and ϕ the phase difference between V and the branch current $I/2$. If the ohmic resistance of the sample, for direct current, is denoted by r_x' and if Δr_f represents the increase in

[2] This is only approximately true since the resistance adjustment changes the phase also to a certain extent.

[3] 1 gilbert = 0.79578 ampere-turn.

resistance due to skin effect at the frequency f, and since the increase of resistance due to hysteresis and eddy current losses is given by the term

$$\frac{\text{total iron losses}}{\left(\frac{I}{2}\right)^2}$$

the apparent resistance, r_x, of the sample is defined by

$$r_x = r_x' + \Delta r_f + \frac{W_c}{\left(\frac{I}{2}\right)^2}$$

$$= r_s + r \qquad (8)$$

where W_c stands for the total core loss in the ferro-magnetic substance. To equate r_x to the quantity (r_s+r) is correct if

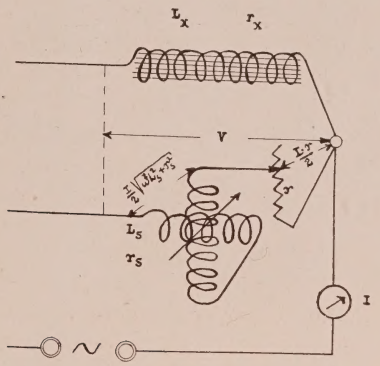

FIGURE 2

we assume that the wire used for the variometer shows no appreciable skin effect, so that its direct current resistance $r' = r$ is equal to the alternating current resistance r''. Within the range of the very high (radio) frequencies such an assumption can not be made, even if ideal twisted wires or ribbons are used, such as described by the author in a previous publication,[2] and the high frequency resistance of the variometer is to be determined by the well known methods if a calibration curve is not available. The magnetizing current I_m which is to be introduced in the equation (7) can be expressed in terms of the effective

[4] A. Hund, "Arbeiten aus dem Elektrotech. Institut der Technischen Hochschule, Karlsruhe," Volume III.

current thru the test sample. For the balanced differential system we obtain

$$I_m = \frac{I}{2} \sin \phi \tag{9}$$

This equation is based on the assumption that the resistance of the windings of the sample is small as compared with the inductive reactance, a requirement which is easily fulfilled, espe-

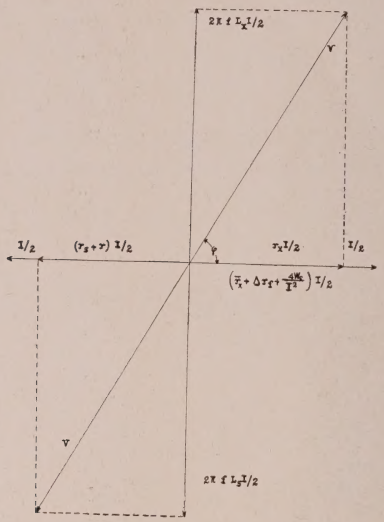

FIGURE 3*

cially at higher frequencies where the wattless components usually become very pronounced. The proof of this approximation may be readily seen from the following consideration:

The total iron loss of the test sample produces an increase in current and decrease in phase displacement, ϕ, between impressed voltage and current under the conditions of constant terminal voltage. That is, the iron acts like a secondary circuit which is coupled to the coil of the sample. This fact is shown in Figure 4 where $A\,B\,C$ denotes the impedance triangle for the sample coil without iron, for which

$\overline{A\,B} = 2\,\pi f L_x$, the inductive reactance

$\overline{B\,C} = r_x'' = r_x' + \Delta\, r_f$, the effective ohmic resistance at the frequency f and

* Wherever an r with a dash (—) over it appears in a figure, it corresponds to r' in the text; similarly an r with a cycle mark ($\sim$) over it corresponds to r'' in the text.

$\overline{A\,C} = Z_x$

$= \sqrt{(2\pi f L_x)^2 + r_x''^2}$, the resistance operator of the coil.

The impedance triangle goes over into the triangle $A'B\,C'$ for the same terminal voltage when the iron core is added, which represents the true conditions of the sample. In this case we have

$\overline{A'B} = 2\pi f\,(L_x - \Delta L_x)$, the inductive reactance

$\overline{BC'} = r_x = r_x'' + \dfrac{W_c}{\left(\dfrac{I}{2}\right)^2}$, the apparent ohmic resistance

$\overline{A'C'} = Z_x'$

$= \sqrt{\left[2\pi f\,(L_x - \Delta L_x)\right]^2 + \left[r_x'' + \dfrac{W_c}{\left(\dfrac{I}{2}\right)^2}\right]^2}$,

the resistance operator of the coil

} for a certain current of definite frequency.

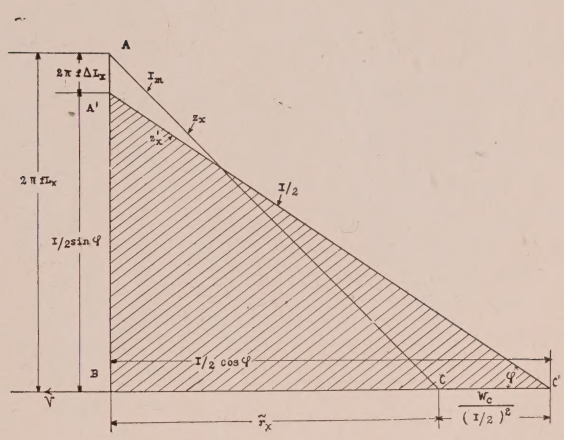

FIGURE 4

The angle $A'C'B$ represents the actual displacement of phase between impressed voltage and current traversing the coil. A little further analytical reasoning will show that the decrement in self inductance due to the presence of the ferro-magnetic substance is given by the expression

$$\Delta L_x = \frac{(2\pi f M)^2 L_2}{r_2^2 + (2\pi f L_2)^2} \qquad (10)$$

and the increment in resistance due to the presence of the ferro-magnetic substance by

$$\frac{W_c}{\left(\frac{I}{2}\right)^2} = \frac{(2\pi f M)^2 r_2}{r_2^2 + (2\pi f L_2)^2} \tag{11}$$

in which case we imagine the iron core to be substituted by a secondary circuit of self inductance L_2 and ohmic resistance r_2. M denotes then the mutual inductance between the coil of the test sample and the fictitious turns of the secondary. We know from the theory of alternating currents that the current triangle is similar to the impedance triangle; which, when applied to our case, means that Figure 4 simultaneously represents the current relations. The scale, of course, would have to be properly selected. Thus the shaded triangle, the case where a ferro-magnetic substance is present, is as follows:

$\overline{A'B} = \frac{I}{2} \sin\phi$, the wattless component } of the current

$\overline{BC'} = \frac{I}{2} \cos\phi$, the watt component } in the coil

$\overline{A'C'} = \frac{I}{2}$, the actual current passing thru the coil of the sample and which is determined from the ammeter reading in the main branch of the differential system.

The triangle $A\,B\,C$ represents the vector diagram for the currents when no iron core is present and the hypothenuse denotes the magnetizing current I_m which is utilized in equation (7) for the evaluation of the magnetizing force H_{max}.

Returning to equation (10) we learn that ΔL_x is only a very small quantity when the term, r_2^2, in the fictitious secondary circuit is large as compared with the term $(2\pi L_2)^2$. This is true in our case to a fair degree of approximation when we assume that the eddy currents are induced in a well subdivided iron core, such as is usually employed in alternating current practice at higher frequencies. With this assumption, the point A and A' may be thought of as coinciding, which leads to a current diagram such as is shown in Figure 5. The base B C' represents, to a certain scale the total power input for the test sample containing a ferro-magnetic core since $C\,C'$ denotes the input due to hysteresis and eddy current loss, and $B\,C$ the input due to copper loss in the turns of the sample. If the resistance of the sample is kept small in comparison with the inductive reactance, as indicated in Figure 5, the magnetizing current,

I_m, is only a little smaller than the wattless component of the true and measurable coil current. This means that for a thinly laminated iron core and using the so-called ideally twisted wire[5] in the test coil of rather low resistance and comparatively high self inductance, the sine component of the branch current passing

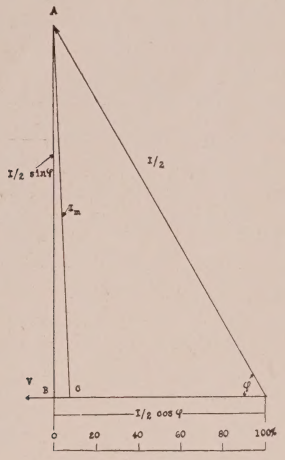

FIGURE 5

thru the sample denotes to a fair degree of approximation the magnetizing current I_m. Returning to equation (9) we find there the magnetizing force, H_{max}, and the magnetic permeability, μ, by the following procedure:

$$\sin\phi = \frac{2\pi f L_x}{\sqrt{r_x^2 + (2\pi f L_x)^2}}$$

$$= \frac{2\pi f L_s}{\sqrt{(r_s+r)^2 + (2\pi f L_s)^2}}$$

then the magnetizing current becomes

$$I_m = \frac{I}{2} \cdot \frac{2\pi f L_s}{\sqrt{(r_s+r)^2 + (2\pi f L_s)^2}} \qquad (10)$$

and according to equation (7) the maximum magnetizing force is defined as

$$H_{max} = 5.57 \frac{N f L_{s\,(henry)} \cdot I_{(amps)}}{l_{(cm)} \sqrt{(r_{s\,(ohm)} + r_{(ohm)})^2 + 2\pi f L_{s\,(henry)})^2}} \text{ gilberts per centimeter} \qquad (11)$$

[5] "Litzendraht."

and the magnetic permeability is given by the relation

$$\mu = \frac{B_{max}}{H_{max}}$$

$$= 1.27 \frac{l_{(cm)} \sqrt{(r_{s\,(ohm)} + r_{(ohm)})^2 + (2\pi f L_{s\,(henry)})^2}}{N^2 \cdot S_{(cm)^2} f} 10^7 \qquad (12)$$

When the wave length, λ, is introduced, as is often customary within the range of radio frequencies, the maximum magnetizing force and magnetic permeability may be determined by the following relations[6]:

$$H_{max} = 16.71 \frac{N \cdot L_{s\,(henry)} I_{(amps)}}{\lambda_{(m)} l_{(cm)} \sqrt{(r_{s\,(ohm)} + r_{(ohm)})^2 + \left(\frac{6\pi \cdot 10^8 L_{s\,(henry)}}{\lambda_{(m)}}\right)^2}} \times 10^8$$

gilberts per centimeter (11*a*)

and

$$\mu = 42.33 \frac{\lambda_{(m)} l_{(cm)} \sqrt{(r_{s\,(ohm)} + r_{(ohm)})^2 + \left(\frac{6\pi \cdot 10^8 L_{s\,(henry)}}{\lambda_{(m)}}\right)^2}}{N^2 S_{(cm)^2}} \times 10^{-3} \qquad (12a)$$

We therefore see that for a standard test ring of given cross sectional area, length of magnetic path and number of turns, the maximum flux density, B_{max}, the maximum resultant field intensity, H_{max}, and the permeability, μ, can be calculated from observed data by means of the three following formulae:

$$B_{max} = k_1 L_{s\,(henry)} I_{(amps)}$$ lines of induction per square centimeter (13)

$$H_{max} = k_2 \frac{f L_{s\,(henry)} I_{(amps)}}{Z_{(ohms)}}$$ gilberts per centimeter (14)

$$\mu = k_3 \frac{z_{(ohms)}}{f} \qquad (15)$$

where z is equivalent to the resistance operator of the test sample and the constants k_1, k_2, and k_3 of a definite dimensioned sample are given by the relations:

$$\left.\begin{aligned} k_1 &= \frac{0.7075 \times 10^8}{N . S_{(cm)^2}} \\ k_2 &= 5.57 \frac{N}{l_{(cm)}} \\ k_3 &= 1.27 \frac{l_{(cm)}}{N^2 S_{(cm)^2}} 10^7 \end{aligned}\right\} \qquad (16)$$

[6] In above formulas, both meters and centimeters are purposely employed, since the wave meters are usually calibrated in meters and the length of the magnetic path is generally measured in centimeters. To express also the self induction in the C. G. S. system was not done since most of the commercial variometers are calibrated in practical units.

A few concluding remarks on the true magnetizing current, I_m, such as utilized for the calculation of the magnetizing force, are given here, before the determination of the core losses is discussed. Such considerations may not be entirely new, altho it seems worth while to add a more detailed analysis in direct application to the differential system, and to derive other expressions with which to calculate the magnetizing force and the permeability.

One should first clearly distinguish between exciting current and magnetizing current, since in ordinary engineering discussions both expressions are often used interchangeably to denote the same quantity, namely, the no-load current of the transformer and the feeding current of a choke coil, respectively. Exciting current of the test sample is the total flow of electricity that passes thru the coil. It is denoted by $I/2$ for the balanced differential system, and is obtained from the ammeter reading of the main branch of the arrangement. This current includes that consumed as Joulean heat losses in the windings of the sample, and also for supplying the losses due to hysteresis and eddy currents. The exciting current may therefore be split up into an energy component, determined by the total loss which is in the vectorial direction of the terminal pressure, V, and into the magnetizing component, I_m, which is in the vectorial direction of the magnetic induction, B, i. e., wattless. Now, if the flux density of the ferro-magnetic medium is within the range of the straight part of the magnetization curve, the magnetizing current, I_m, will vary according to a sine law when the flux is sinusoidal. But in case of higher saturations for which the flux density rises beyond the straight part of the saturation curve (for instance, beyond the knee), the magnetizing current becomes distorted. Consequently, the exciting current which a sinusoidal impressed E. M. F. will establish in the turns of our test sample no longer varies according to a sine law. This can be seen from Figure 6 where the instantaneous current values, $i/2$, of the test coil, such as have been obtained from the hysteresis loop, are plotted against the time. It is to be noted that the ohmic drop in the winding is considered as being small enough to be neglected in this representation, so that it is possible to assume that the induced E. M. F. is at all times equal and opposite to the impressed terminal voltage, V, of the sample. The figure clearly shows that the exciting current, $i/2$, required to produce a sinusoidal flux density wave is unsymmetrical with respect to its maximum ordinate. The maximum flux density occurs at

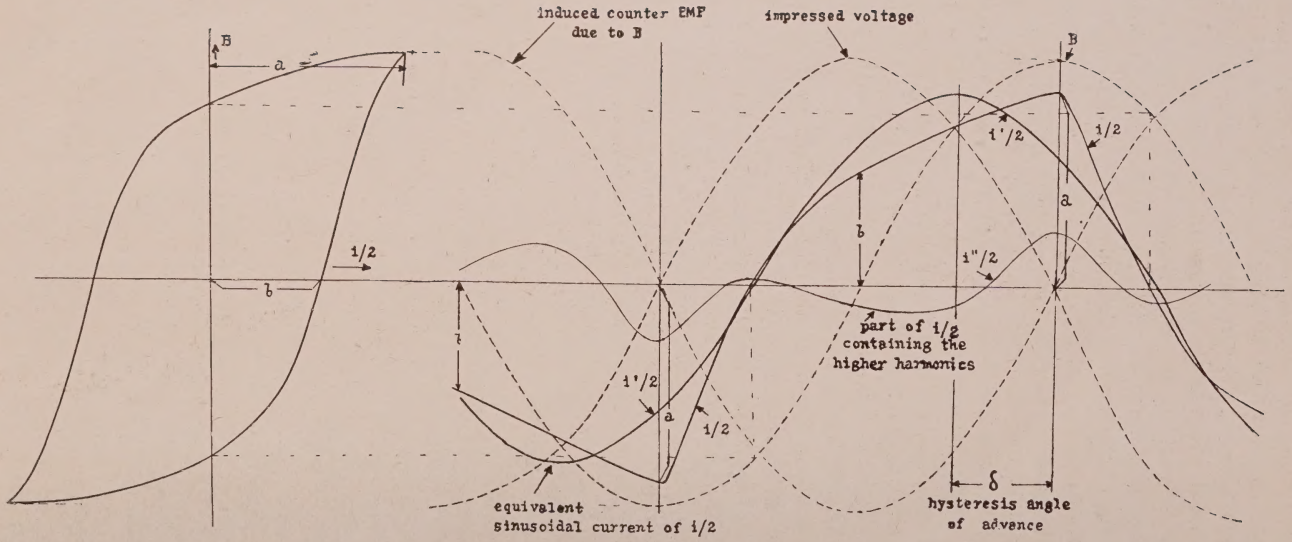
induced counter EMF
due to B
impressed voltage
B
a
i/2
b
i'/2
a
i/2
equivalent
sinusoidal current of i/2
B
i'/2
i/2
a
b
i"/2
part of i/2
containing the
higher harmonics
δ
hysteresis angle
of advance

FIGURE 6

the same time that the exciting current reaches its maximum; that is to say, this current is 90 time-degrees ahead of the induced E. M. F. and generally about 90 time-degrees behind the impressed voltage, whereas the intersection with the zero line indicates a considerable lead with respect to the zero value of the flux density. The wave of the exciting current of commercial frequency up to the highest frequencies, such as employed in radio telegraphy and telephony, is usually distorted by the presence of higher harmonics of a very pronounced triple harmonic. As is indicated above, the distortion is chiefly due to the magnetizing current, I_m, and is caused on account of the curved part of the B-H curve. With transformers, this distortion is greatly diminished by the load current which, when large enough, makes insignificant the well defined distorting component of the exciting current. In our case, however, where a closed magnetic circuit is often employed, the distortion is very pronounced and an analysis of the exciting circuit is accordingly of interest. One way of studying the distorted exciting current is to split it up into those components which are in phase with the induced E. M. F. (and are often called the hysteresis and power components), and one component 90 time-degrees behind it (that is, in phase with the magnetic induction, B, and representing the magnetizing current). Such a resolution of the exciting current is shown in Figure 7, which represents a typical case. It is readily seen

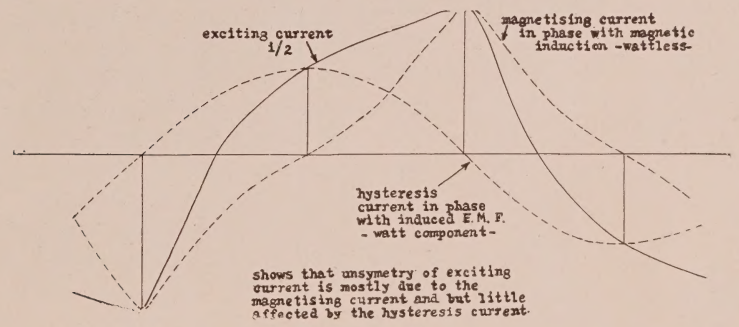

FIGURE 7

that the distortion of the exciting current, such as is obtained from the hysteresis loop, is chiefly influenced by the magnetizing component as a consequence of the non-proportionality of the

B-H relation, whereas the hysteresis current wave is a good approximation a sine curve. (The reader who would like to pursue this subject in more detail will find a very interesting treatment in Dr. Steinmetz's "Alternating Current Phenomena.") In Figure 6, the exciting current is, however, resolved into a first harmonic component of the same power and effective value as that of the exciting current curve and into a component containing the higher harmonics. The latter component, which is composed of the higher harmonics, is wattless with respect to the sinusoidal applied voltage of fundamental frequency and consequently the effective watt component, $I'_w/2$, of the equivalent sine wave, $i'/2$, denotes the total power component of the exciting current and is equivalent to I_h, the hysteresis current. The magnetizing current, I_m, is then constituted of the wattless component, $I'_{wL}/2$, of the equivalent sine wave, $i'/2$, and the effective value $I''/2$ of the curve, $i''/2$, containing the higher harmonics, chiefly of triple frequency. This is made plainer by the vector diagram of Figure 8, which shows the construction of the effective value, $I/2$, of the exciting current, $i/2$; which, by assumption, is also equal to the effective value of the equivalent sine curve. The vector, $I/2$, makes the angle, δ, with the true magnetizing current. The same is called the angle of hysteretic phase advance, and denotes the angle by which the first harmonic of the exciting current leads the sinusoidal wave of the flux density, B. The effective value of the measurable coil current of the test sample is therefore given by the relation

$$\frac{I}{2}=\sqrt{\left[\frac{I'_w}{2}\right]^2+\left[\left(\frac{I'_{wL}}{2}\right)^2+\left(\frac{I''}{2}\right)^2\right]^2} \qquad (17)$$

and the effective value of the true magnetizing current entering the equation (7) becomes

$$\begin{aligned} I_m &= \sqrt{\left[\frac{I'_{wL}}{2}\right]^2+\left[\frac{I''}{2}\right]^2} \\ &= \sqrt{\left[\frac{I}{2}\right]^2-\left[\frac{I'_w}{2}\right]^2} \end{aligned} \qquad (18)$$

which, translated into technical language, states that the power component demagnetizes the iron core. This can be made plainer by drawing the hysteresis component, $-I_h$, equal and opposite to, I_h. Then the vectors, $-I_h$ and $I/2$, constitute the magnetizing current, I_m. The diagram shows furthermore that for zero hysteresis effect, the magnetizing current would be identical with

the exciting current, $I/2$, altho the wave would be still unsymmetrical due to the varying permeability, μ.

The power, W_h, dissipated because of the hysteresis can be determined by this method, as is shown in a later paragraph. The effective current $I/2$ is obtained from the ammeter reading of the balanced arrangement and the applied terminal voltage, V, is found from the relation:

$$V = \frac{I}{2}\sqrt{r_x^2 + (2\pi f L_x)^2}$$
$$= \frac{I}{2}\sqrt{[r + r_s]^2 + [2\pi f L_s]^2} \qquad (19)$$

The magnetizing current is then determined according to the procedure

$$W_h = V \cdot \frac{I}{2} cos\ (90 - \delta)$$
$$= V \cdot \frac{I}{2} sin\ \delta$$
$$= V \cdot \frac{I'_w}{2} \qquad (20)$$

introduced in equation (18)

$$I_m = \sqrt{\left[\frac{I}{2}\right]^2 - \left[\frac{W_h}{V}\right]^2} \qquad (21)$$

This equation, however, is based on the assumption that the ohmic drop in the test sample is negligibly small and the losses due to eddy currents are accordingly ignored. The first assumption may be readily satisfied by employing so-called "ideal"-twisted wire of low resistance, while the effect due to eddy currents has to be taken into account, especially when taking readings within the range of radio frequencies. It is known that the eddy currents, like magnetic hysteresis, cause the phase of the current to advance; the angle of which phase advance can be calculated from its sine, which is defined by the ratio of the absolute admittance of the circuit to the eddy current conductance. For well laminated iron cores, the distortion of the current wave may be kept very small, as can be demonstrated by investigating the hysteresis loops of different laminated samples by means of a Braun tube. The distortion of the hysteresis loop, due to eddy currents, is caused because they act like secondary circuits and consequently their magnetic fields counteract the main field of the coil. For this reason, in the case of a fixed magnetizing force, the total flux density is smaller when eddy currents are

present than without it. Hysteresis loops such as were found in the splendid researches of Prof. Max Wien[7] and others show this effect very plainly. Furthermore, the loops show rounded corners and are more inclined to the axis of the magnetizing force at higher frequencies, altho the area of the loop seems to change but little as the frequency increases. Another important in-

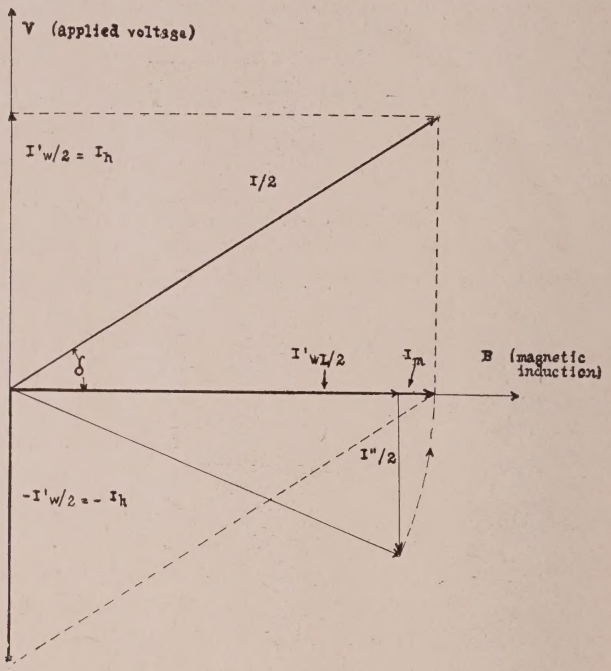

FIGURE 8

fluence of the eddy currents on the hysteresis loss, which is especially pronounced at higher frequencies, is that the lines of magnetic induction are not uniformly distributed over the cross section of the iron core.

Nevertheless, the measurements described in the above paragraph are not any more difficult when the effect of the eddy currents is taken into account; for the flux density wave (and along with it the eddy currents) also follow a sine law with a sinusoidal terminal voltage. The eddy currents simply increase the watt

[7] M. Wien, "Ann. der Physik," 1898.

and wattless components of the equivalent sine wave of the exciting current on account of the larger exciting current. This means that the general formula of equation (21) is given by the expression

$$I_m = \sqrt{\left[\frac{I}{2}\right]^2 - \left[\frac{W_c}{V}\right]^2} \tag{22}$$

and, according to a derivation given in a later paragraph, the true magnetizing current may be found from

$$I_m = \sqrt{\left[\frac{I}{2}\right]^2 - \left[\frac{\left(\frac{I}{2}\right)^2 \Delta r_c}{\frac{I}{2}\sqrt{r_x^2 + (2\pi f L_x)^2}}\right]^2}$$

or

$$I_{m\,(amps)} = \frac{I_{(amps)}}{2}\sqrt{1 - \frac{(\Delta r_{c\,(ohm)})^2}{(r_{(ohm)} + r_{s\,(ohm)})^2 + (2\pi f L_{s\,(henry)})^2}} \tag{23}$$

which leads to the expressions

$$H_{max} = 0.8875\,\frac{N \cdot I_{(amps)}}{l_{(cm)}}\sqrt{1 - \frac{(\Delta r_{c\,(ohm)})^2}{(r_{(ohm)} + r_{s\,(ohm)})^2 + (2\pi f L_{s\,(henry)})^2}} \quad \text{gilberts per centimeter} \tag{24}$$

and

$$\mu = 0.798\,\frac{l_{(cm)}\,L_{s\,(henry)}\,10^8}{N^2\,S_{(cm)^2}\sqrt{1 - \frac{(\Delta r_{c(ohm)})^2}{(r_{(ohms)} + r_{s\,(ohm)})^2 + (2\pi f L_{s\,(henry)})^2}}} \tag{25}$$

We therefore see that for a standard test ring of given cross sectional area, length of magnetic path and number of turns, the maximum resultant field intensity, H_{max}, and the magnetic permeability, μ, can be calculated from observed data by means of the following formulas:

$$H_{max} = k_4\,I_{(amps)}\sqrt{1 - \left(\frac{\Delta r_{c\,(ohm)}}{z_{(ohm)}}\right)^2} \quad \text{gilberts per centimeter} \tag{26}$$

and

$$\mu = k_5\,\frac{L_{s\,(henry)}}{\sqrt{1 - \left(\frac{\Delta r_{c\,(ohm)}}{z_{(ohm)}}\right)^2}} \tag{27}$$

where z represents again the resistance operator of the test sample and constants k_4 and k_5 are given by the expressions

$$\left.\begin{aligned} k_4 &= 0.8875\,\frac{N}{l_{(cm)}} \\ \text{and} \qquad k_5 &= 0.798 \times 10^8\,\frac{l_{(cm)}}{N^2\,S_{(cm)^2}} \end{aligned}\right\} \tag{28}$$

Determination of Iron Losses

If the magnetic field of a coil is replaced by a ferro-magnetic field, the number of lines of induction is increased, which in turn necessitates an increase in the coefficient of self induction of the coil. Since the flux does not increase as the current traversing the coil on an iron core, the coefficient of self finduction is not a constant for a certain frequency, but is a function of the current. Because the iron core is a consumer of energy, the ohmic resistance of the coil will apparently increase (equation (8)). This means that the quantity, r_x, also depends on the current. The differential arrangement therefore is a ready means to determine the iron losses. The procedure is simply this:

Measure the resistance, r_x', of the test sample without iron by means of direct current. Adjust the resistance of the variometer combination to the same value by regulating the series resistance, r. Then apply the desired high frequency current of a definite frequency, f, to the differential system and increase r in the variometer branch by the quantity, Δr_f; that is, until an amplitude adjustment is attained. It is also advisable simultaneously to balance the phases of the two differential branches by making, $L_s = L_x$, since then the amplitude balance is more easily obtainable under such conditions. The additional resistance, Δr_f, then represents the increase of the ohmic resistance due to skin effect, which produces the additional loss, $\Delta r_f \left(\frac{I}{2}\right)^2$. Now insert the iron core in the coil, adjust the phase by again varying the standard self inductance (that is, until, $L_{s_c} = L_{x_c}$), where L_{x_c} denotes the effective coefficient of self induction of the test coil in the presence of iron for a certain wave length and a definite current value. Then add the resistance, Δr_c, until a complete balance is reached. The quantity, Δr_c, stands then for the increase of resistance due to core loss for a certain current and wave length, which leads to the following relations:

$$\begin{aligned} W_c &= \left[\frac{I}{2}\right]^2 \Delta r_c \\ &= W_h + W_e \\ &= \eta \cdot f\, v\, B_{max}^{\alpha}\, 10^{-7} + \xi\, f^2\, d^2\, v\, B_{max}^{\beta}\, 10^{-14} \end{aligned} \qquad (29)$$

where W_c, W_e, W_h denote: the total core loss, eddy current loss, hysteresis loss in watts, η the hysteresis coefficient, ξ the eddy current coefficient, v the volume of the iron in cm.^{3} B_{max} the maximum number of lines of induction per cm.,^{2} d the thickness of the iron laminations in cm. The ex-

ponents α and β can be determined by observations at different flux densities. The separation of the hysteresis and the eddy current losses may be carried on in the usual way, when observations are taken for the same flux density at two different frequencies, f_1 and f_2, using the following expressions

$$\left.\begin{aligned} \frac{W_{c_1}}{f_1} &= \eta\, v\, B_{max}^{\alpha}\, 10^{-7} + \xi f_1\, d^2\, v\, B_{max}^{\beta}\, 10^{-14} \\ &= K_1 B_{max}^{\alpha} + K_2 f_1 B_{max}^{\beta} \\ &= A + f_1 D \end{aligned}\right\} \quad (30)$$

and

$$\left.\begin{aligned} \frac{W_{c_2}}{f_2} &= \eta\, v\, B_{max}^{\alpha}\, 10^{-7} + \xi f_2\, d^2\, v\, B_{max}^{\beta}\, 10^{-14} \\ &= K_1 B_{max}^{\alpha} + K_2 f_2 B_{max}^{\beta} \\ &= A + f_2 D \end{aligned}\right\}$$

where, A, denotes the hysteresis loss in watts per cycle and, $f \cdot D$, the eddy current loss in watts per cycle. Hence

$$\left.\begin{aligned} A &= \frac{\dfrac{W_{c_1} \cdot f_2}{f_1} - \dfrac{W_{c_2} \cdot f_1}{f_2}}{f_2 - f_1} \\ &= 0.333.. \frac{W_{c_1} \cdot \lambda_1^2 - W_{c_2} \cdot \lambda_2^2}{\lambda_1 - \lambda_2} \times 10^{-8} \end{aligned}\right\} \quad (31)$$

and

$$\left.\begin{aligned} D &= \frac{\dfrac{W_{c_1}}{f_1} - \dfrac{W_{c_2}}{f_2}}{f_1 - f_2} \\ &= 0.111\,.. \frac{W_{c_1} \lambda_1^2 \lambda_2 - W_{c_2} \lambda_1 \lambda_2^2}{\lambda_2 - \lambda_1} \times 10^{-16} \end{aligned}\right\}$$

where the wave length, λ, is again expressed in meters and W_{c_1} and W_{c_2} in watts.

Practical Hints On a Proper Differential Arrangement—A Discussion of Disturbances and Means of Overcoming Them

As can be seen from the introduction, the similarity and symmetrical arrangement of the two differential coils are a most important feature for the proper design of the transformer, because dissymmetry of the windings affects the inductance and resistance as well as the capacity phenomena of the primary coils with respect to the secondary. When these factors are not absolutely the same for each primary coil the phase of the radio frequency currents will be shifted unequally. All these conditions are complied with by the application of the so-called "ideal"-twisted wires. Approximately six turns in each primary coil with a diameter of about six inches (15 cm.) are recommended,

in order to avert any unnecessarily large transformer losses. The same number of turns may be conveniently used for the secondary coil of the air transformer altho it is advisable to compute the number required for a particular zero current indicator.

Other disturbances present themselves because of the effect of the magnetic fields of the test sample and variometer in the secondary circuit of the transformer, and sometimes it is entirely impossible to cause the effects of the differential field to disappear completely. These influences are, however, overcome by arranging the test and comparison apparatus in such a way as to make their induction upon the transformer and secondary circuit a minimum. By choosing long leads on the one side (connecting to the transformer) the above-mentioned disturbances are practically eliminated.

Furthermore, it should be noted that inductive and capacity effects of the different parts of the arrangement with respect to each other are very pronounced within the range of radio frequency currents such as employed in radio telegraphy and telephony. Numerous investigations have shown that an accurate equalization of the differential system is practically impossible without extreme precautions. For this reason a double cable (bifilar) enclosed in a grounded brass tube was used for all leads such as the mains which connect the apparatus to the radio frequency source and the leads connecting the zero current indicator with the differential transformer. One joint of the differential arrangement is also grounded (Figure 1) in order to cut down unnecessary leakage currents.

Further, it is interesting to study the case in which the resistance used for compensating the loss of the coil shows appreciable self induction and capacity effects. Consider the differential arrangement of Figure 9 as follows: The inserted series resistances, r_1 and r_2, each had a certain amount of self induction, ΔL_1 and ΔL_2, as well as a definite value of capacity, ΔC_1 and ΔC_2. Comparing the test sample with the variometer and these resistances, we obtain for the balance condition

$$r_x+r_2+j\left\{\omega\,[L_x+\Delta L_2]-\frac{1}{\omega\,\Delta C_2}\right\}$$
$$=r_s+r_1+j\left\{\omega\,[L_s+\Delta L_1]-\frac{1}{\omega\,\Delta C_1}\right\} \qquad (32)$$

hence
$$r_x-r_s=r_1-r_2 \qquad (33)$$

and the imaginary part gives the ratio

$$\frac{L_x}{L_s}=1+\frac{1}{L_s}\left[(\Delta L_1-\Delta L_2)+\frac{1}{{}^2\omega}\left(\frac{\Delta C_1-\Delta C_2}{\Delta C_1\cdot\Delta C_2}\right)\right] \qquad (34)$$

which shows that the difference of self induction, $\Delta L_1 - \Delta L_2$, has to be small in comparison with the effective self induction of the test sample. This condition is practically satisfied by the use of short length of manganin or constantin wire for the resistances r_1 and r_2. It is seen from equation (33) that the measurement of the difference of the resistance of test sample and variometer is not affected by any inductive or capacity effects of the

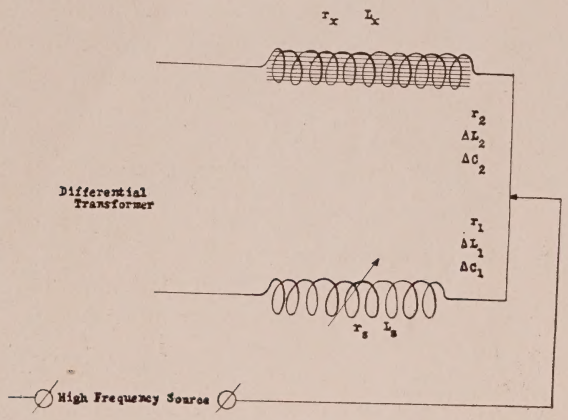

FIGURE 9

inserted balancing resistances, r_1 and r_2. This is an essential advantage of the differential method in comparison with the usually applied bridge arrangements, for the same inductive and capacity influences of the resistance introduce considerable errors at higher frequencies unless the bifilar bridge of Giebe is applied. The latter requires, however, an exact knowledge of the capacity and inductance of the leads, and is hardly available for measurements such as are met with in radio telegraphy. The influence of inductive effects of the resistances upon the loss adjustment, in case a bridge arrangement were used, may be seen fron Figure 10 and the following deductions. We then have as a general condition of balance

$$\frac{r_x + j\,\omega L_x}{r_s + j\,\omega L_s} = \frac{r_2 + j\,\omega\,\Delta L_2}{r_1 + j\,\omega\,\Delta L_1} \tag{35}$$

and separating the real and imaginary parts,

$$r_x r_1 - r_s \cdot r_2 = \omega^2 [L_x\,\Delta L_1 - L_s\,\Delta L_2] \tag{36}$$

$$L_x r_1 - L_s r_2 = r_s \Delta L_2 - r_x \Delta L_1 \tag{37}$$

or

$$\frac{r_x}{r_s} = \frac{r_2}{r_1} + (2\pi f)^2 \left[\frac{L_x \Delta L_1 - L_s \Delta L_2}{r_1 r_s}\right] \tag{38}$$

$$\frac{L_x}{L_s} = \frac{r_2}{r_1} - \left[\frac{r_x \Delta L_1 - r_s \Delta L_2}{L_s r_1}\right] \tag{39}$$

Equation 38 shows clearly that the resistance adjustment is considerably affected at higher frequencies when inductive effects

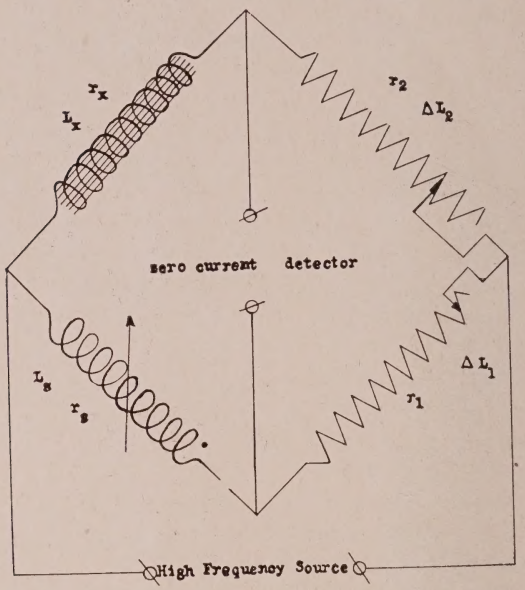

FIGURE 10

of r_1 and r_2 present themselves. Moreover, the bridge method is not advantageous since four branches act inductively on each other. Furthermore, three terms are to be varied for balance instead of only two and the protection of the detector circuit would cause grave difficulties.

The ordinary thermo-couple arrangement and barreter system are convenient expedients as zero current detectors, altho for very delicate readings the thermo-cross bridge is to be recommended.

Before concluding, it might be of interest to investigate somewhat the effects of the disturbing capacity with respect

to the body of the observer and the like. For this purpose imagine that the test sample be balanced against the comparison standard. The zero current indicator will, however, only indicate a definite minimum at very high frequencies, even if resonance is established in the detector circuit by means of a condenser. Assume, further, that all the above precautions are employed and that the leads of the zero current detector are well protected against disturbances such as mentioned above. Yet the zero current detector will indicate a certain flux interlinked with the secondary circuit of the differential transformer. The cause of the disturbance can only be based upon capacity influences, which can be proved experimentally. For instance, when the observer touched different parts of the differential arrangement the telephone receiver (for this class of investigations, an oscillatory detector arrangement was employed as zero-current indicator) gave different sounds at the minimum. By putting the hand on one of the secondary terminals of the tranformer, the minimum was better. This phenomenon is due to charges and discharges causing a leakage current, flowing from the primary to the secondary turns of the differential transformer and such a stray current flowing from the turns of the telephone receiver thru the metal case and the hand of the observer to the ground. The primary coil, the secondary coil and the turns of the telephone receiver are regarded each as one pole of a condenser. This assumption can be made, as there will be not a strict mathematical treatment of this case; but this assumption is simply used for the proper interpretation of the cause of the above phenomena. Suppose, V_3, is the potential of the primary turns, V_2, that of the secondary coil, V_1, the potential of the turns of the telephone receiver, and the body of the observer has the potential V_o. Assuming further that C_{32} is the capacity of the condenser formed by the primary and secondary coil of the differential transformer, and C_{10} the capacity of the condenser formed by the turns of the telephone receiver and the body of the observer. Then, from the primary to the secondary turns of the transformer a charging current, $[V_3-V_2]\,\omega\,C_{32}$, flows, of which a certain part, $[V_1-V_o]\,\omega\,C_{10}$, flows thru the hand to ground. Suppose we touch one of the secondary terminals of the transformer, that is, that the same is brought to the potential V_o. Consequently the second stray current disappears and the first one becomes, $(V_3-V_o)\omega\,C_{32}$. It might be believed, that the increase, $\{[V_3-V_o]-[V_3-V_2]\}\,\omega C_{32}$ would affect the telephone receiver more and would not dim-

inish the sound. But if we bear in mind, that the body and therefore the potential V_o) is connected with one of the secondary terminals of the transformer, it is understood that practically most of the stray current will be led thru the observer to the ground. It would be a wrong expedient against these disturbances to ground one of the secondary terminals, as that would only diminish the sensitiveness of the arrangement. Instead the writer used a tube of glass for handling the slide resistance and a cord for turning the coils of the standard variometer. By this means the current, $(V_1 - V_o)\omega C_{10}$ could be made exceedingly small. (It is to be noted that for very precise measurements the telephone receiver is to be replaced by a galvanometer.) The first capacity current flowing from the primary to the secondary turns affected the telephone receiver much more. In order to overcome this disturbance the writer put a copper cylinder around the secondary turns. The cylinder consisted of enamelled copper wire. Along a longitudinal line the insulation was removed and all turns of the cylinder connected to ground. On an opposite longitudinal line, the protecting cylinder was cut in order that the damping action of the transformer might not be increased too much.

SUMMARY: The method described gives a ready means for determining the magnetizing force, the corresponding flux density, and permeability at any wave length whatever. In taking a series of readings for different ampere-turns and at a definite wave length, we may obtain
(a) the magnetization curve,
(b) the permeability-ampere-turns curve.
Since the suggested arrangement applies to any practically available wave length we have a convenient means to compare the B_{max}-values for very long wave lengths with the corresponding values determined at higher frequencies, and thus obtain a clear insight into the skin action of an iron core.

In a similar way the permeability-wave length curve may be found for a definite number of ampere turns. The method simultaneously determines either the total losses of the test sample or the losses due to direct current resistance, skin effect of the conductor, hysteresis and eddy currents in the iron core separately, and there can be obtained the
(a) watts/unit volume-wave length curve for a certain number of ampere turns,
(b) watts/unit volume-flux density curve at a constant wave length, and
(c) watts/unit volume-thickness of laminations curve for a constant wave length and a constant number of ampere turns; which enables the investigator to ascertain all the conditions which are necessary for determining the desired properties of any radio frequency apparatus containing a ferro-magnetic medium.

Moreover, by means of equations (14) and (15) one is able to experimentally investigate the dependance of the magnetizing force and permeability on the frequency.

LIST OF SYMBOLS USED

a	Hysteresis exponent.
β	Eddy current exponent.
B_{max}	Maximum magnetic flux density, number of lines of magnetic induction per square centimeter.
$C_{10}, C_{32}, \Delta C_1, \Delta C_2,$	Capacities in farads.
d	Thickness of iron lamination in centimeters.
δ	Hysteretic angle of advance.
E	Effective induced E. M. F. of N turns of the test sample.
E_{av}	Average induced voltage of a single turn.
η	Hysteresis constant.
F	Form factor.
f	Frequency.
Φ_{max}	Maximum flux traversing the test coil.
Φ_t	Flux traversing the test coil at any time t.
ϕ	Displacement of phase between terminal voltage and current of the test sample.
H_{max}	Maximum resultant field intensity in gilberts per centimeter.
I	Current flowing to the differential system.
$I''/2$	Effective current value of the component of the exciting current of the test sample containing the higher harmonics.
I_h	Effective value of hysteresis current.
I_m	Effective value of magnetizing component of exciting current.
$I'_W/2$	Effective value of the energy component of the equivalent sine wave of the exciting current.
$I'_{WL}/2$	Wattless component of the equivalent sine wave.
$i/2$	Instantaneous value of the exciting current.
$i'/2$	Instantaneous value of the equivalent sine wave.
$i''/2$	Instantaneous value of the component of the exciting current containing all higher harmonics.
$j=\sqrt{-1}$	The imaginary unit.
$K_1, K_2, k_1, k_2, k_3, k_4, k_5$	Constants.

$L_s, L_{sc}, L_x, L_{xc}, L_2, \Delta L_1, \Delta L_2, \Delta L_x$	Coefficients of self induction in henrys.
l	Length of magnetic path in centimeters.
λ	Wave length in meters.
μ	Magnetic permeability.
N	Number of turns of the test sample.
r	Series resistance in ohms.
r_x'	Direct current resistance of test sample.
r_x''	Alternating current resistance of test sample.
r_x	Resistance of test sample under any condition.
Δr_c	Increase of the resistance of the test sample caused by core loss.
Δr_f	Increase of the resistance at the frequency f caused by skin effect.
S	Cross sectional area of iron core in square centimeters.
T	Period of radio frequency current.
V, V_1, V_2, V_3	Terminal voltages.
v	Volume of iron core in cubic centimeters.
W_c	Total core loss in watts.
W_{c1}	Total core loss in watts at a frequency f_1.
W_{c2}	Total core loss in watts at a frequency f_2.
W_e	Core loss due to eddy currents.
W_h	Core loss due to hysteresis.
ω	Angular velocity of radio frequency current.
ξ	Eddy current constant.
z	Resistance operator of variometer-resistance combination.
z_x	Resistance operator of test sample when no iron is present.
z_x'	Resistance operator of test sample when iron is present.

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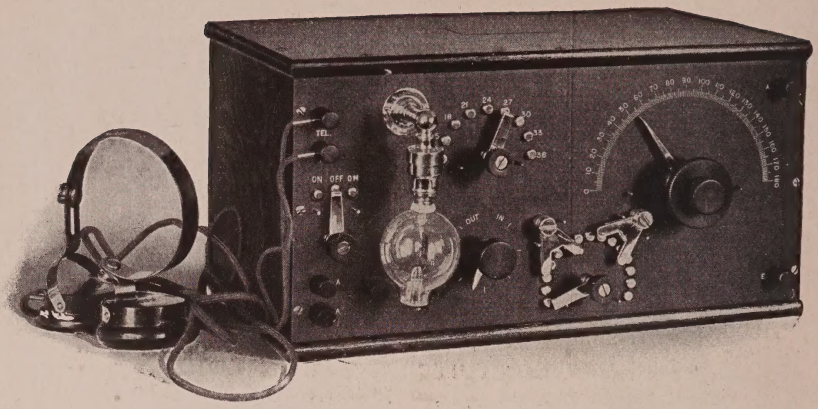

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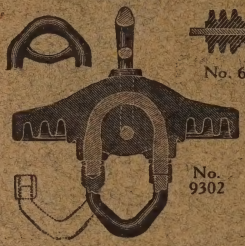
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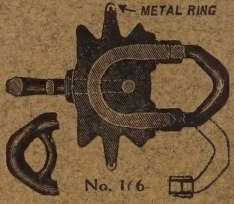

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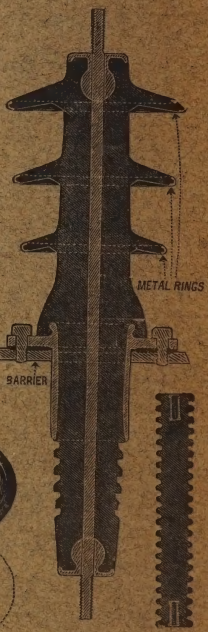

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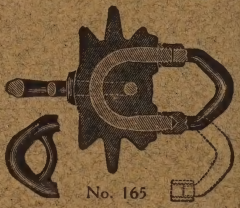
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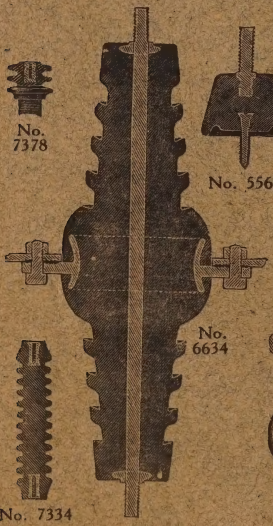
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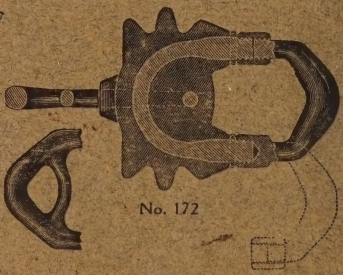
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